وزارة التعليم العالي والبحث العلمي

جامعة الفرات الأوسط التقنية

المعهد التقني – الديوانية

حقيبة رقم (1)

وحده نمطية لدراسة

(*Simple Stresses in Machines Parts*)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة: - دراسة أنواع الإجهادات البسيطة لأجزاء المكائن ودراسة منحني (الإجهاد – الانفعال) وتحديد مفهوم عمليات التصميم على هذا المخطط ودراسة عامل الأمان ودوره في عمليات التصميم

إعداد

المدرس

فائق حامد جبوري

(1) Simple Stresses in Machines Parts

1 – 1 Introduction

In engineering practice, the machine parts are subjected to various forces which may be due to either one or more of the followings:-

1- Energy transmitted.
2- Weight of machine.
3- Frictional resistance.
4- Inertia of reciprocating part.
5- Change of temperature.
6- Lack of balance of moving parts.

The different forces acting on a machine part produce various types of stresses.

1 – 2 Load

It is defined as any external force acting upon a machine part. The following four types of the load are important from the subject point of view.

1- Dead or steady load. A load is said to be a dead or steady load, when it does not change in magnitude or direction.
2- Live or variable load. A load is said to be a live or variable load, when it changes continually.
3- Suddenly applied or shock loads. A load is be a suddenly applied or shock load, when it is suddenly applied or removed.
4- Impact load. A load is said to be an impact load, when it is applied with some initial velocity.

Note:- A machine part resists a dead load more easily than a live load more easily than a shock load.

1 – 3 Stress

When some external system of forces or loads act on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as unit stress or simply a stress. It is denoted by a Greek letter sigma (σ). Mathematically,

$$\text{Stress,} \quad \sigma = \frac{F}{A}$$

Where F – Force or load acting on a body, and

A – Cross – sectional area of the body.

In S.I units, the stress is usually expressed in Pascal (Pa) such that (1 Pa = 1 N/m^2). In actual practice, we use bigger units of stress i.e., megapascal (MPa) and gigapascal (GPa), such that

$1 \text{ MPa} = 1\times 10^6 \ N/m^2 = 1 \ N/mm^2$

$1 \text{ GPa} = 1\times 10^9 \ N/m^2 = 1 \ KN/mm^2$

1 – 4 strain

When a system of forces or loads act on a body, it undergoes some deformation per unit length is known as unit strain or simply a strain. It is denoted by a Greek letter epsilon (ε). Mathematically,

$$\text{Strain,} \quad \varepsilon = \frac{\delta l}{l} \qquad \text{or} \qquad \delta l = \varepsilon . l$$

Where δl – Change in length of the body, and

l – Original length of the body.

1 – 5 Tensile Stress and Strain

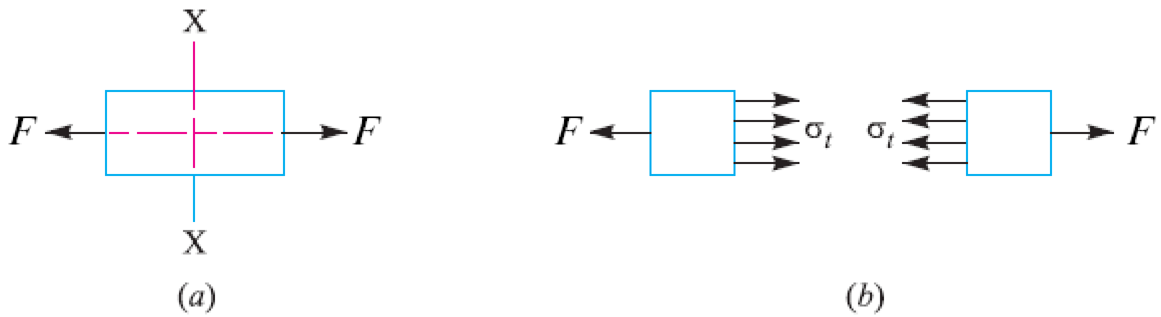


Fig. (1 – 1) Tensile stress and strain.

When the body is subjected to two equal axial pulls F (also called tensile load) as shown in Figure (1 – 1 . a), then the stress induced at any section of the body is known as tensile stress as shown in Figure (1 – 1. b). A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as tensile strain.

Let F – Axial tensile force acting on the body,
A – Cross-sectional area of the body,
l – Original length, and
δl – Increase in length.

$\therefore$ Tensile stress, $\sigma_t = \dfrac{F}{A}$

and tensile strain, $\varepsilon_t = \dfrac{\delta l}{l}$

1 – 6 Compressive Stress and Strain

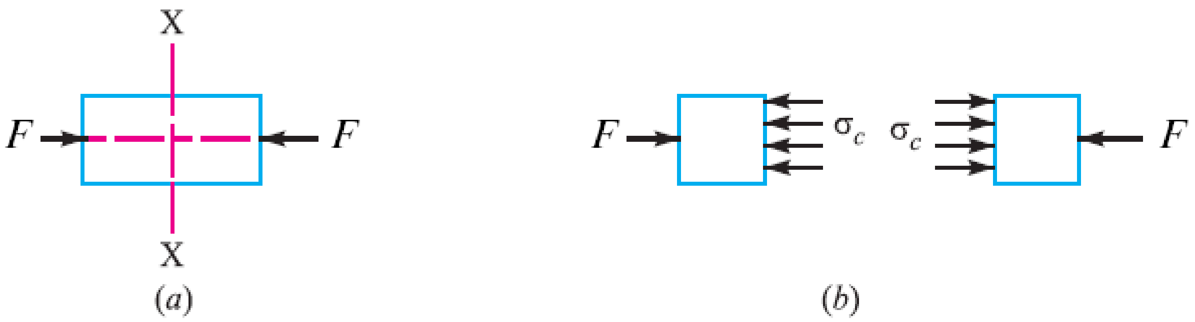


Fig. (1 – 2) Compressive stress and strain.

When the body is subjected to two equal and opposite axial pushes F (also called compressive load) as shown in Figure (1 – 2 . a), then the stress induced at any section of the body is known as compressive stress as shown in Figure (1– 2. b). A little consideration will show that due to the compressive load, there will be an increase in cross-sectional area and a decrease in length of the body. The ratio of the decrease in length to the original length is known as compressive strain.

Let F – Axial tensile force acting on the body,
A – Cross-sectional area of the body,
l – Original length, and
δl – Increase in length.

$\therefore$ Compressive stress, $\sigma_c = \dfrac{F}{A}$

and Compressive strain, $\varepsilon_c = \dfrac{\delta l}{l}$

Note:- In case of tension or compressive, the area involved is at right to the external force applied.

1 – 7 Young's Modulus or Modulus of Elasticity

Hooke's law – states that when a material is loaded within elastic limit, the stress is directly proportional to strain, i.e.

$\sigma \; \alpha \; \varepsilon$ or $\sigma = E\varepsilon$

$$E = \frac{\sigma}{\varepsilon} = \frac{\dfrac{F}{A}}{\dfrac{\delta l}{l}} \qquad\qquad E = \frac{F \times l}{A \times \delta l}$$

Where (E) is a constant of proportionality known as Young's modulus or modulus of elasticity. In S.I. units, it is usually expressed in GPa i.e., GN/m^2 or kN/mm^2. It may be noted that Hooke's law holds good for tension as well as compression.

The following table shows the values of modulus of elasticity or Young's modulus (E) for the materials commonly used in engineering practice.

Material	Modulus of elasticity (E) in GPa i.e. GN/m^2 or kN/mm^2
Steel and Nickel	200 to 220
Wrought iron	190 to 200
Cast iron	100 to 160
Copper	90 to 110
Brass	80 to 90
Aluminum	60 to 180
Timber	10

1 – 8 Shear Stress

When a body is subjected to two equal and opposite forces, acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called shear stress.

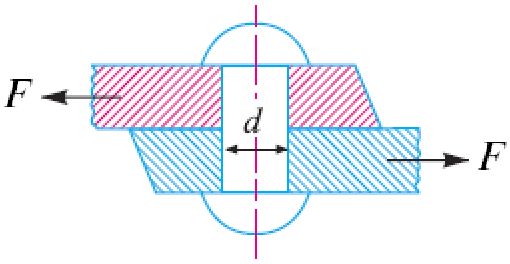


Fig. (1 – 3)

Shear stress, τ = Tangential force / Resisting area

$$\tau = \frac{F}{A} = \frac{F}{\pi r^2} \quad \text{or} \quad = \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2}$$

1 – 9 Bearing Stress

A localised compressive stress at the surface of contact between two members of a machine part, that are relatively at rest is known as bearing stress or crushing stress. The bearing stress is taken into account in the design of riveted joints, cotter joints, knuckle joints etc.

σ_c or σ_b , $\qquad \sigma_c = \dfrac{F}{d \times t \times n}$

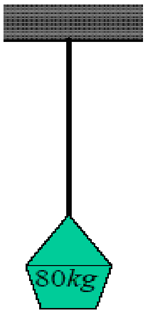
Where: d – Diameter of the rivet,
t – Thickness of the plate,
$d \times t$ – Projected area of the rivet, and
n – Number of rivets per pitch length in bearing or crushing.

Example (1):- An (80 kg) lamp is supported by a single electrical copper cable (wire). What is the diameter of cable if the maximum allowable stress for copper is ($50 N/mm^2$) ?

Solution:- $A = \frac{\pi d^2}{4} \Rightarrow d^2 = \frac{4A}{\pi} \Rightarrow d = \sqrt{\frac{4A}{\pi}}$

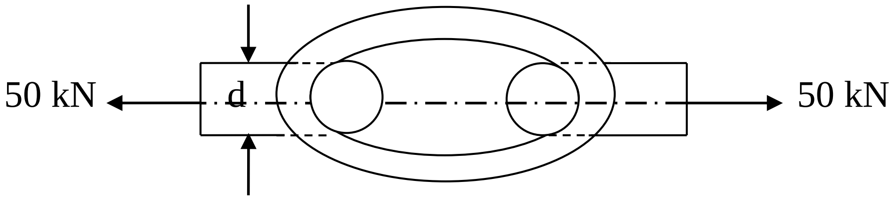
$F = mg$

$\sigma = \frac{F}{A} \Rightarrow A = \frac{F}{\sigma} \Rightarrow A = \frac{80 \times 9.81}{50} = 15.696 \ mm^2$



$\therefore \ d = \sqrt{\frac{4 \times 15.696}{\pi}} = 4.47 \ mm$

Example (2):- A coil chain of a crane required to carry a maximum load of (50 kN), is shown in Figure below. Find the diameter of the link stock, if the permissible tensile in the link material is not to exceed (75 MPa).



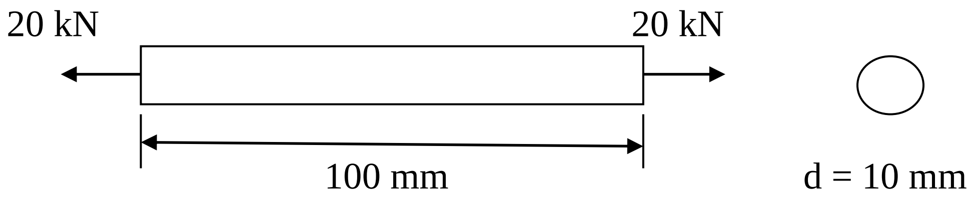
Solution:- Give: F = 50 kN = 50 × 10^3 N, σ_t = 75 MPa = 75 N / mm^2

Area, $A = \frac{\pi}{4} d^2$ = 0.7854 d^2

$\sigma_t = \frac{F}{A} \Rightarrow A = \frac{F}{\sigma_t}$

$0.7854 d^2 = \frac{50 \times 10^3}{75} \Rightarrow d = 29.13 mm$

Example (3):- Determine the stress and extension of the (10 mm) diameter bar shown in the figure. The modulus of elasticity for the bar material (E = 200 GPa).



Solution:- Given: $F = 20\ kN$, $l = 100\ mm$, $d = 10\ mm$,

$E = 200\ GN/m^2 = 200\ kN/mm^2$

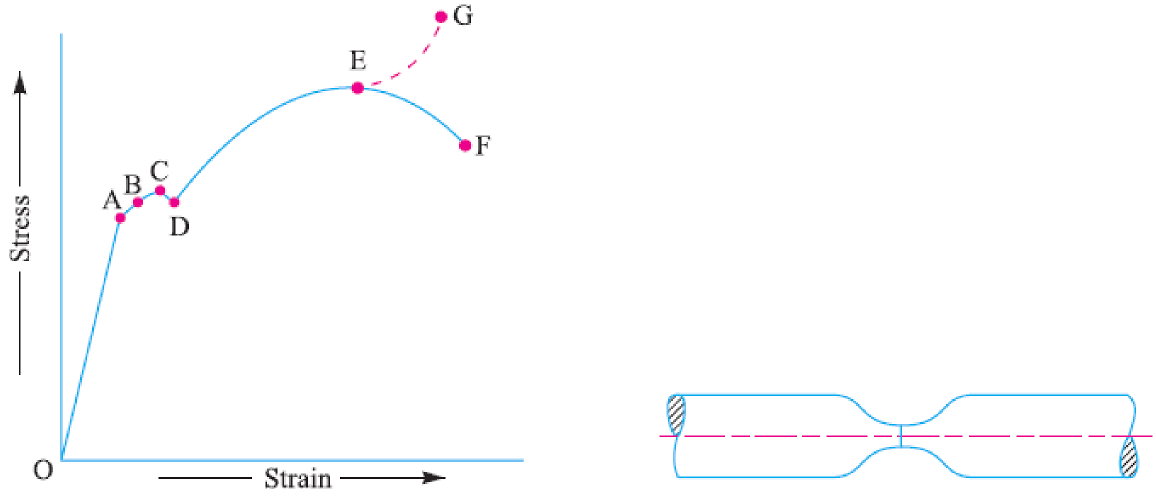
Area $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(10)^2 = 78.54\ mm^2$

Stress $\sigma_t = \frac{F}{A} = \frac{20 \times 10^3}{78.54} = 254.64\ MPa$

Extension (δl)

$$E = \frac{F \times l}{A \times \delta l} \Rightarrow \delta l = \frac{F \times l}{A \times E} = \frac{20 \times 10^3 \times 100}{78.54 \times 200 \times 10^3} = 0.1237 mm$$

1 – 10 Stress – strain diagram



(*a*) (*b*) Shape of specimen after elongation.

Fig. (1 – 4)

In designing various parts of a machine, it is necessary to know how the material will function in service. For this, certain characteristics or properties of the material should be known. The mechanical properties mostly used in mechanical engineering practice are commonly determined from a standard tensile test. This test consists of gradually loading a standard specimen of a material and noting the corresponding values of load and elongation until the specimen fractures. The load is applied and measured by a testing machine. The stress is determined by dividing the load values by the original cross-sectional area of the specimen. The elongation is measured by determining the amounts that two reference points on the specimen are moved apart by the action of the machine. The original distance between the two reference points is known as gauge length. The strain is determined by dividing the elongation values by the gauge length.

The values of the stress and corresponding strain are used to draw the stress-strain diagram of the material tested. A stress-strain diagram for a mild steel under tensile test is shown in Fig. above .The various properties of the material are discussed below:

1- Proportional limit.

We see from the diagram that from point *O* to *A* is a straight line, which represents that the stress is proportional to strain. Beyond point *A*, the curve slightly deviates from the straight line. It is thus obvious, that Hooke's law holds good up to point *A* and it is known as ***proportional limit***. It is defined as that stress at which the stress-strain curve begins to deviate from the straight line.

2- Elastic limit.

It may be noted that even if the load is increased beyond point *A* upto the point *B*, the material will regain its shape and size when the load is removed. This means that the material has elastic properties up to the point *B*. This point is known as

elastic limit. It is defined as the stress developed in the material without any permanent set.

Note: Since the above two limits are very close to each other, therefore, for all practical purposes these are taken to be equal.

3- Yield point.

If the material is stressed beyond point *B*, the plastic stage will reach *i.e*. on the removal of the load, the material will not be able to recover its original size and shape. A little consideration will show that beyond point *B*, the strain increases at a faster rate with any increase in the stress until the point *C* is reached. At this point, the material yields before the load and there is an appreciable strain without any increase in stress. In case of mild steel, it will be seen that a small load drops to *D*, immediately after yielding commences. Hence there are two yield points *C* and *D*. The points *C* and *D* are called the ***upper*** and ***lower yield points*** respectively. The stress corresponding to yield point is known as ***yield point stress.***

4- Ultimate stress.

At *D*, the specimen regains some strength and higher values of stresses are required for higher strains, than those between *A* and *D*. The stress (or load) goes on increasing till the point *E* is reached. The gradual increase in the strain (or length) of the specimen is followed with the uniform reduction of its cross-sectional area. The work done, during stretching the specimen, is transformed largely into heat and the specimen becomes hot. At *E*, the stress, which attains its maximum value is known as ***ultimate stress.*** It is defined as the largest stress obtained by dividing the largest value of the load reached in a test to the original cross-sectional area of the test piece.

5- Breaking stress.

After the specimen has reached the ultimate stress, a neck is formed, which decreases the cross-sectional area of the specimen, as shown in Fig. (1 – 4 . *b*). A little consideration will show that the stress (or load) necessary to break away the specimen, is less than the maximum stress. The stress is, therefore, reduced until the specimen breaks away at point *F*. The stress corresponding to point *F* is known as ***breaking stress.***

Note : The breaking stress (*i.e*. stress at *F* which is less than at *E*) appears to be somewhat misleading. As the formation of a neck takes place at *E* which reduces the cross-sectional area, it causes the specimen suddenly to fail at *F*. If for each value of the strain between *E* and *F*, the tensile load is divided by the reduced crosssectional area at the narrowest part of the neck, then the true stress-strain curve will follow the dotted line *EG*. However, it is an established practice, to calculate strains on the basis of original cross-sectional area of the specimen.

Pre test (1)

Circle the correct answer:-

1- Hooke's law holds good up to:-

a- yield point. b- elastic limit.

c- plastic limit. d- breaking point.

2- The ratio of linear stress to linear strain is called:-

a- modulus of elasticity b- modulus of rigidity.

c- bulk modulus d- Poisson's ratio.

3- When the material is loaded within elastic limit, then the stress is -------- to strain.

a- equal. b- directly proportional. C- inversely proportional.

4- When a hole of diameter (*d*) is punched in a metal of thickness (*t*), then the force required to punch a hole is equal to:-

a- $d \times t \times \tau_u$ b- $\pi \times d \times t \times \tau_u$

c- $\frac{\pi}{4} d^2 \times \tau_u$ d- $\frac{\pi}{4} d^2 \times t \times \tau_u$

where:- τ_u = Ultimate shear strength of the material of the plate.

1 – 11 Working Stress

When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the working stress or design stress. It is also known as safe or allowable stress.

Note:- By failure it is not meant actual breaking of the material. Some machine parts are said to fail when they have plastic deformation set in them, and they no more perform their function satisfactory.

1 – 12 Factor of Safety (*n*)

It is defined, in general, as the ratio of the maximum stress to the working stress. mathematically,

$$\text{Factor of safety } (n) = \frac{Maximum\ stress}{Working\ stress}$$

In case of ductile materials e.g., mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

$$\text{Factor of safety } (n) = \frac{Yield\ point\ \ stress}{Working\ stress}$$

In case of brittle materials e.g., cast iron, the yield point is not well defined as for ductile materials. Therefore the factor of safety for brittle materials is based on ultimate stress.

$$\therefore\ \text{Factor of safety } (n) = \frac{Ultimate\ stress}{Working\ or\ design\ stress}$$

This relation may also be used for ductile materials.

Note: The above relations for factor of safety are for static loading.

1 – 13 Selection of Factor of Safety

The selection of a proper factor of safety to be used in designing any machine component depends upon a number of considerations, such as the material, mode of manufacture, type of stress, general service conditions and shape of the parts. Before selecting a proper factor of safety, a design engineer should consider the following points :

1- The reliability of the properties of the material and change of these properties during service .
2- The reliability of test results and accuracy of application of these results to actual machine parts .
3- The reliability of applied load .
4- The certainty as to exact mode of failure .
5- The extent of simplifying assumptions .
6- The extent of localised stresses .
7- The extent of initial stresses set up during manufacture .
8-The extent of loss of life if failure occurs , and
9- The extent of loss of property if failure occurs. Each of the above factors must be carefully considered and evaluated. The high factor of safety results in

unnecessary risk of failure. The values of factor of safety based on ultimate strength for different materials and type of load are given in the following table:-

Material	Steady load	Live load	Shock load
Cast iron	5 to 6	8 to 12	16 to 20
Wrought iron	4	7	10 to 15
Steel	4	8	12 to 16
Soft materials and alloys	6	9	15
Leather	9	12	15
Timber	7	10 to 15	20

Pre test (2)

Circle the correct answer:-

1- The ratio of the ultimate stress to the design stress is known as:-

a- elastic limit. b- strain.

c- factor of safety. d- bulk modulus.

2- The factor of safety for steel and for steady load is:-

a- 2 b- 4 c- 6 d- 8

3- A localized compressive stress at the area of contact between two members is known as:-

a- tensile stress. b- bending stress.

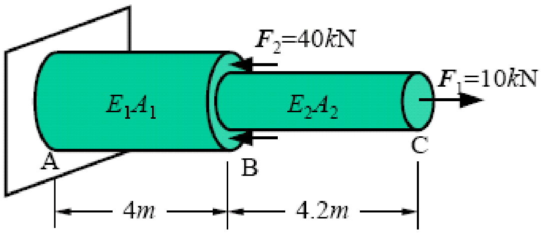
c- bearing stress. d- shear stress.

4- An aluminium member is designed based on:-

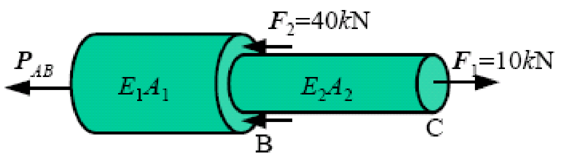
a- yield stress b- elastic limit stress

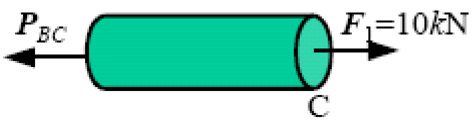
c- proof stress d- ultimate stress

Example (4):- The composite bar shown in the figure is made of two segments, AB and BC, having cross-sectional areas of $A_1 = 200mm^2$ and $A_2 = 100mm^2$. Their Young's modules are ($E_1 = 100GPa$) and ($E_2=210GPa$) respectively. Find the total displacement at the right end.



Solution:-





$$\delta_{AC} = \frac{F_{AB}L_{AB}}{A_{AB}E_{AB}} + \frac{F_{BC}L_{BC}}{A_{BC}E_{BC}}$$

$$\delta_{AC} = \frac{-30\times10^3\times4}{200\times10^{-6}\times100\times10^9} + \frac{10\times10^3\times4.2}{100\times10^{-6}\times210\times10^9}$$

$$\delta_{AC} = -0.006 + 0.002 \quad \Rightarrow \quad \delta_{AC} = -0.004mm$$

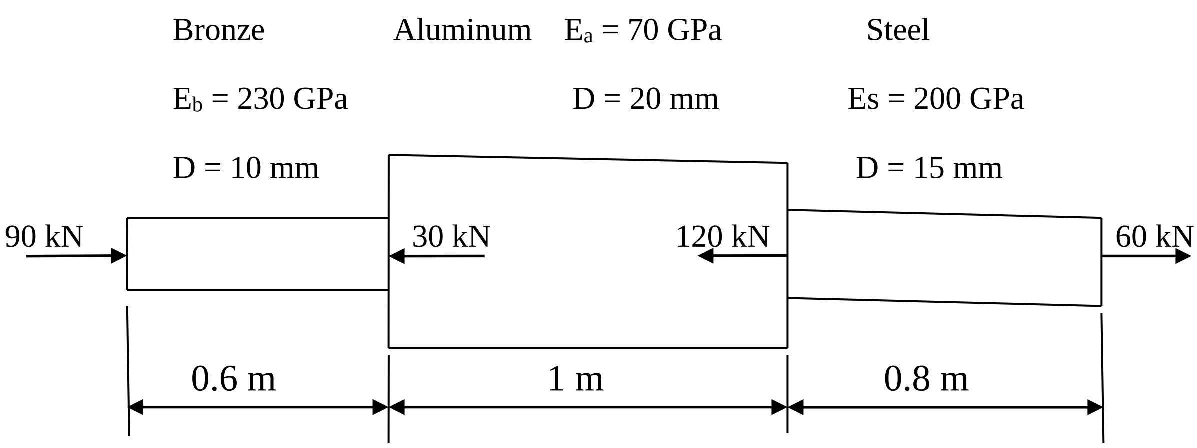
Post test

1-Determine the elongation of the steel bar (1 m) long and (1.5 cm^2) cross-sectional area, when subjected to a pull of (15 kN). Take E = 200 GPa ?

2 – A brass rod (2 cm) diameter and (1.5 m) long is subjected to an axial pull of (4 kN). Find the stress, strain and elongation of the rod, if the modulus of Elasticity for the brass is 85 GPa?

3- Aluminum tube is rigidity fasted between a bronze rod and steel rod as shown in the figure. Determine:-

1- The deformation in each material ? 2- The stress in each material ?



Key

Pre test (1)

Questions	Answers
1-	b
2-	a
3-	b

Pre test (2)

Questions	Answers
1-	c
2-	b
3-	c
4-	a

Post test

Questions	Answers
1-	$\delta l = 3.33\times 10^{-3}$ mm
2-	$\sigma = 12.732$ MPa, $\varepsilon = 1.4979 \times 10^{-4}$, $\delta l = 0.2246$ mm
3-	$\delta l_b = 2.989$ mm, $\delta l_{AL} = 2.728$ mm, $\delta l_{st} = 1.358$ mm
	$\sigma_b = 113.097$ MPa, $\sigma_{AL} = 190.98$ MPa, $\sigma_{st} = 339.53$ MPa

References:-

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2- Machinery's Handbook
ERIK OBERG, FRANKLIN D. JONES, HOLBROOK L. HORTON, AND HENRY H. RYFFEL

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جامعة الفرات الأوسط التقنية

المعهد التقني – الديوانية

حقيبة رقم (2)

وحده نمطية لدراسة (*Fastening*)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة مفهوم الربط باللولب, أنواعه, استخداماته, حساباته

إعداد

المدرس

فائق حامد جبوري

(2) Fastening

2 – 1 Types of fastenings:

1 – Temporary fastenings:- such as: bolts, nuts, screws, keys and pins.
2 – Permanent fastenings: - such as: rivets, welding …. etc.

Different parts of structures, machines or other engineering products are joined together by means of fastenings.

2 – 2 Screwed joints

A screwed joint is mainly composed of two elements: bolt and nut. Screwed joints are widely used where the machine parts are required to be readily connected or disconnected without damage to the machine or the fastening.

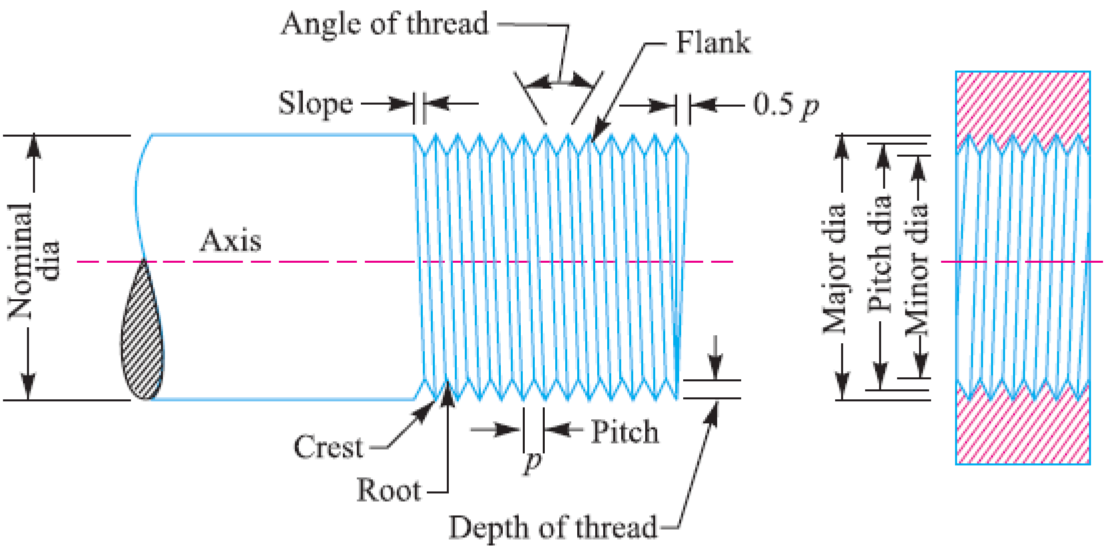


Fig. (2 – 1) Terms used in screw threads.

2 – 3 Advantages and Disadvantages of Screwed Joints

Following are the advantages and disadvantages of the screwed joints.

Advantages

1- Screwed joints are highly reliable in operation.
2- Screwed joints are convenient to assemble and disassemble.
3- A wide range of screwed joints may be adopted to various operating conditions.
4- Screws are relatively cheap to produce due to standardization and highly efficient manufacturing processes.

Disadvantages

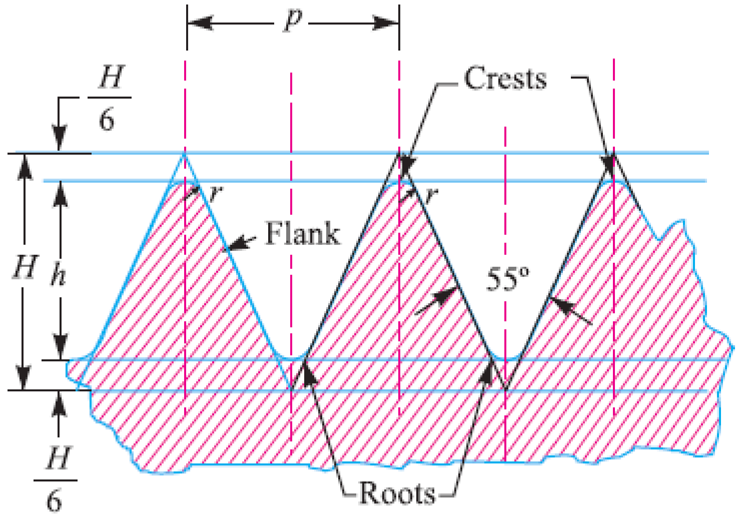
The main disadvantage of the screwed joints is the stress concentration in the threaded portions which are vulnerable points under variable load conditions.
Note : The strength of the screwed joints is not comparable with that of riveted or welded joints.

2 – 4 Forms of screw threads:

The following are the various forms of screw threads:

1- British standard whit worth (B. S. W.) thread:

These threads are found bon bolts and screwed fastenings for special purpose. And used for steel and iron pipes and tubes carrying fluids, in external pipe threading, the threads are specified by the bore of the pipe.

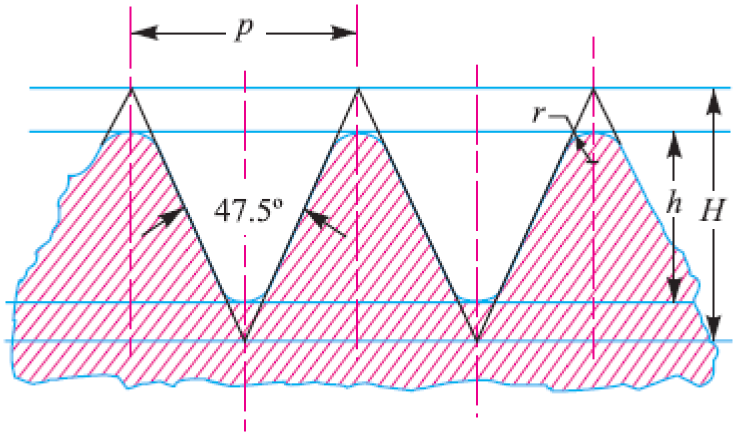


$H = 0.96\,p$; $h = 0.64\,p$; $r = 0.1373\,p$

Fig. (2 – 2) British standard whit worth (B.S.W) thread.

2- British Association (B.A.) thread :

These threads are used for instruments and other precision work.



$H = 1.13634\,p$; $h = 0.6\,p$; $r = 0.18083\,p$

Fig. (2 – 3) British association (B.A.) thread.

3- American National standard (U.S.)thread:

These threads are used for general purposes e.g. on bolts, nuts, screws and tapped holes.

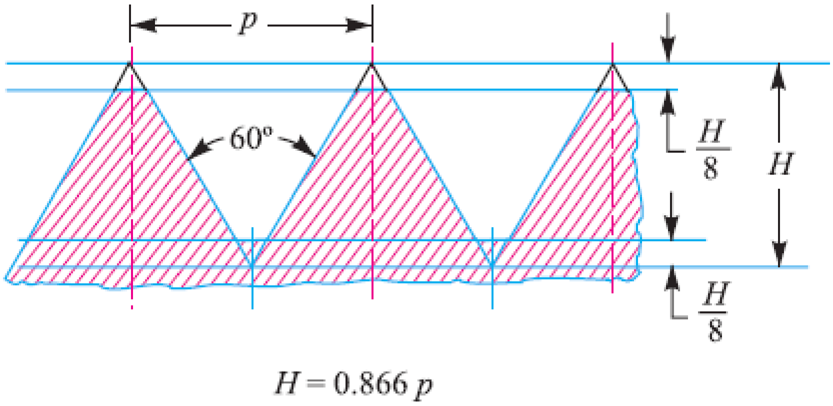


Fig. (2 – 4) American national standard thread.

4- Unified standard thread:

The three countries i.e., Great Britain, Canada and United States. This thread has rounded crests and roots, used for general purposes e.g. on bolts, nuts, screws.

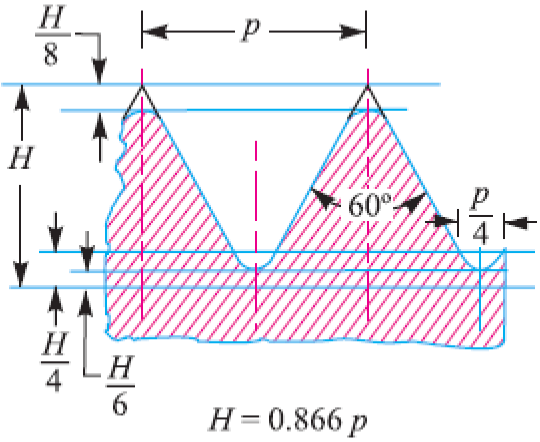


Fig. (2 – 5) Unified standard thread.

5- Square thread:

Because of their high efficiency, are widely used for transmission of power in either direction such type of threads are usually found on the feed mechanisms of machine tools, valves, spindles, screw jacks.

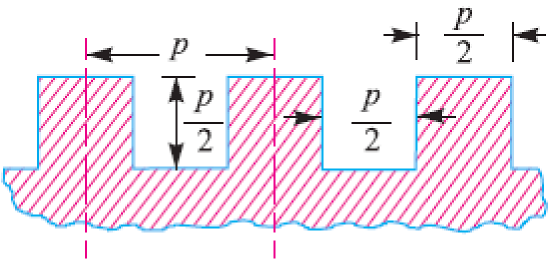


Fig. (2 – 6) Square thread.

6- Acme thread:

It is a modification of square thread. It is much stronger than square thread and can be easily produced. These threads are frequently used on screw cutting lathes, brass valves cock and bench vices.

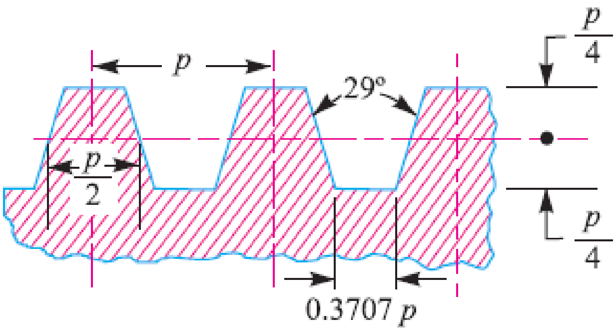


Fig. (2 – 7) Acme thread.

7- Knuckle thread:

It is also a modification of square thread. These threads are used for rough and ready work. They are usually found on railway carriage couplings, hydrants, necks of glass bottles and large and large moulded insulators used in electrical trade.

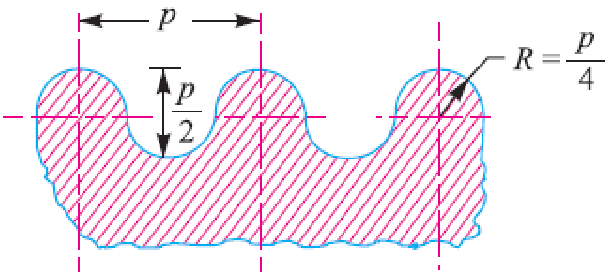


Fig. (2 – 8) Knuckle thread.

8- Buttress thread:

It is used for transmission of power in one direction only. The spindles of bench vices are usually provided with buttress thread, because it has low frictional resistance.

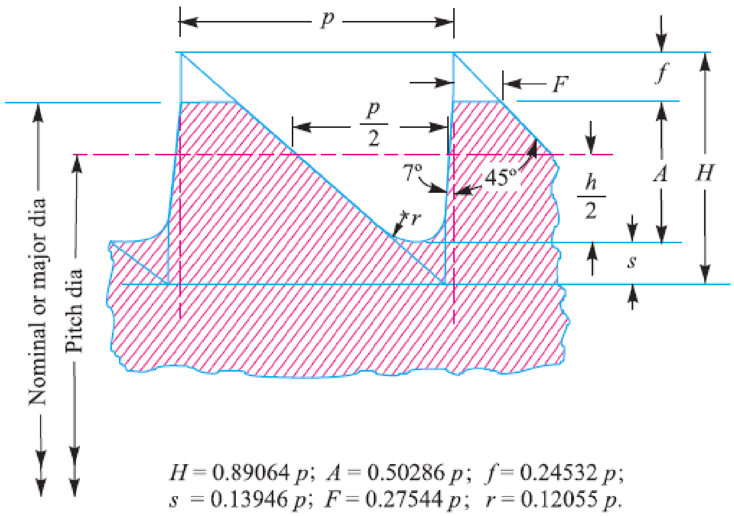


Fig. (2 – 9) Buttress thread.

9- Metric thread:

It is an Indian standard thread and is similar to B.S.W. thread , it has an included angle of (60^o)instead of (55^o).

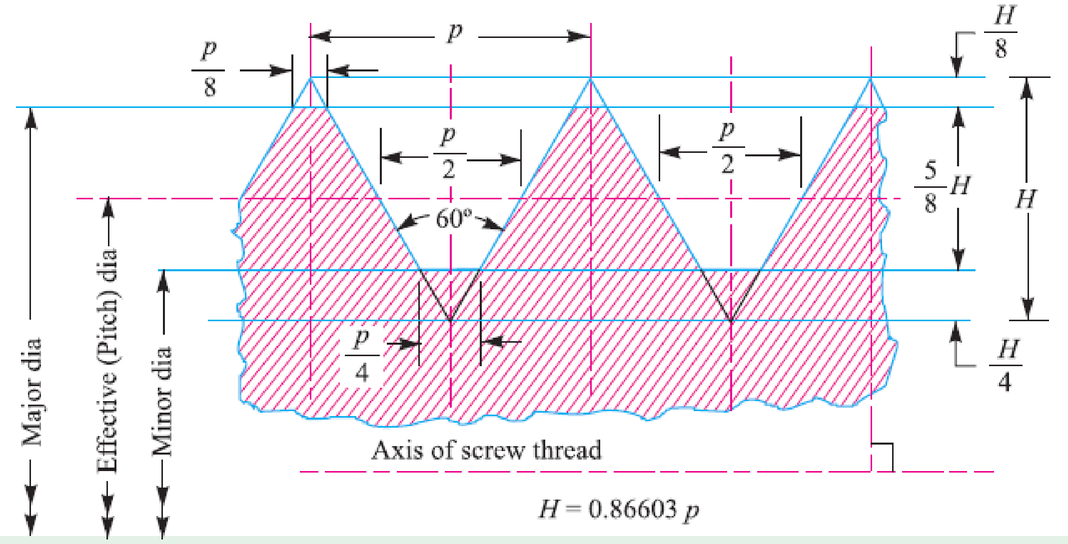


Fig. (2 – 10) Basic profile of the thread.

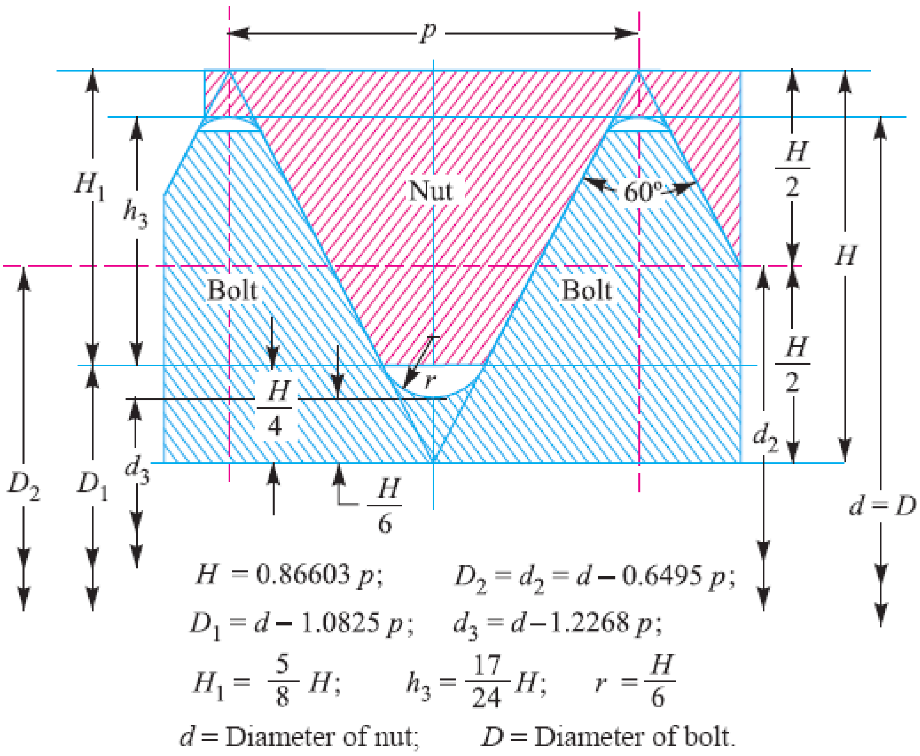


Fig. (2 – 11) Design profile of the nut and bolt.

2 – 5 Common types of Screw Fastening

1- Through bolts

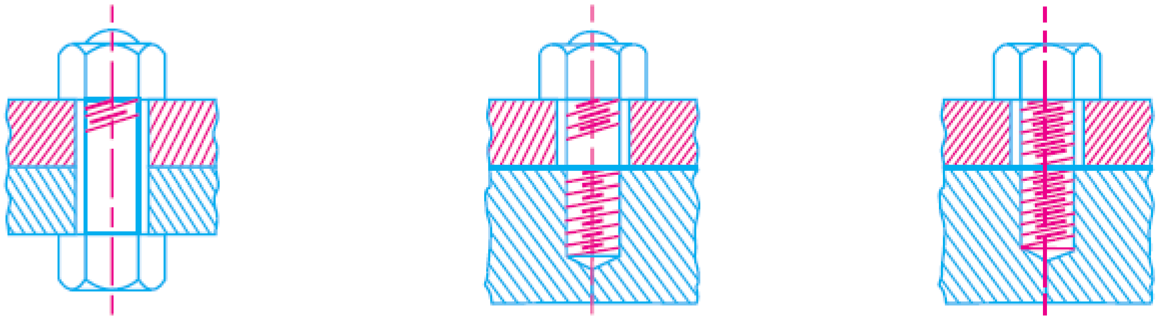
Use may be known as machine bolts, carriage bolts, automobile bolts, eye bolts etc.

2- Tap bolts

A tap bolt or screw differs from a bolt. It is screwed into a tapped hole of one of the parts to be fastened without the nut.

3- Studs

Used instead of tap bolts for securing various kinds of covers *e.g.* covers of engine and pump cylinders, valves, chests etc.

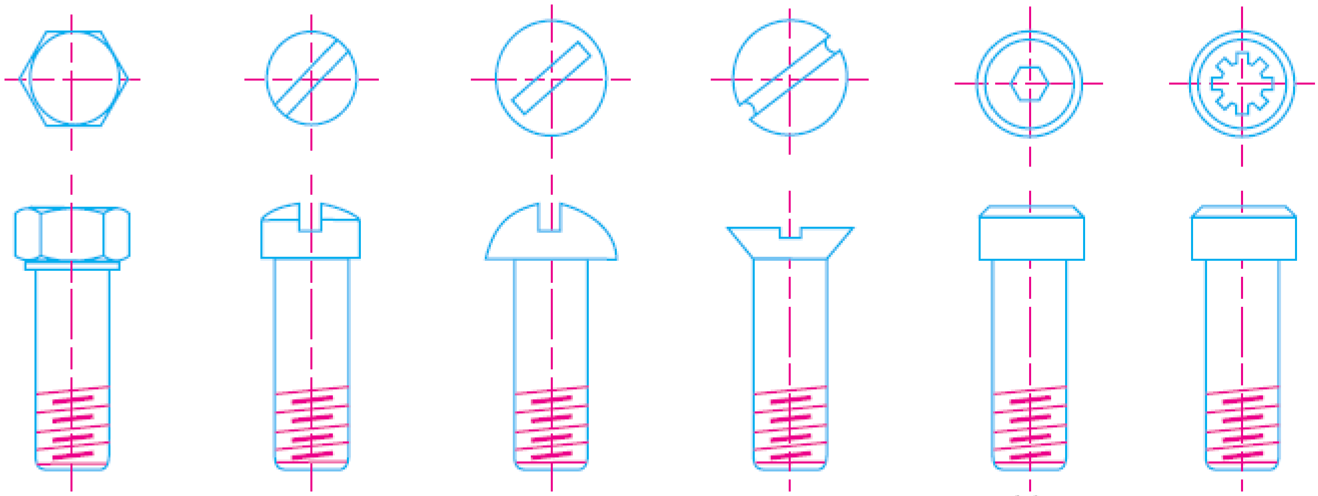


(*a*) Through bolt. (*b*) Tap bolt. (*c*) Stud.

Fig. (2 – 12)

4- Cap screws

The cap screws are similar to tap bolts except that they are of small size and a variety of shapes of heads are available as shown in Fig. 2 – 13.



(*a*) (*b*) (*c*) (*d*) (*e*) (*f*)

(*a*) Hexagonal head; (*b*) Fillister head; (*c*) Round head; (*d*) Flat head; (*e*) Hexagonal socket; (*f*) Fluted socket.

Fig. (2 – 13) Types of cap screws.

5- Machine screws

These are similar to cap screws with the head slotted for a screw driver. These are generally used with a nut.

6- Set screws

These are used to prevent relative motion between the two parts. They may be used instead of key to prevent relative motion between a hub and a shaft in light power transmission members. They may also be used in connection with a key, where they prevent relative axial motion of the shaft, key and hub assembly.

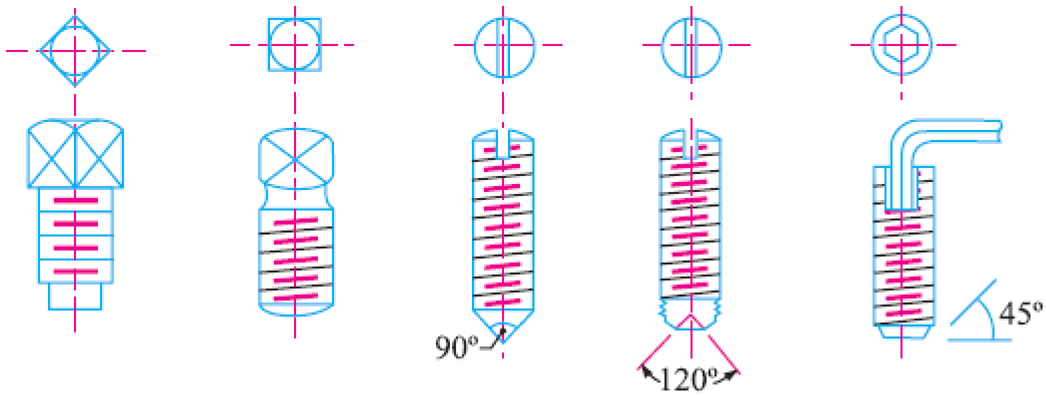


Fig. (2 – 14) Set screws.

Pre test (1)

Circle the correct answer:-

1- The largest diameter of an external or internal screw thread is known as:-
a- minor diameter. b- major diameter.
c- pitch diameter. d- none of these.

2- The pitch diameter is the ------------- diameter of an external or internal screw thread:-
a- effective. b- smallest. c- largest.

3- A screw is specified by its:-
a- major diameter. b- minor diameter.
c- pitch diameter. d- pitch.

4- The railway carriage coupling have:-
a- square threads. b- acme threads.
c- knuckle threads. d- buttress threads.

2 – 6 Stresses in screwed fastening due to static loading

The following stresses in screwed fastening due to static loading are important from the subject point of view.

1- Internal stresses due to screwing up forces.
2- Stress due to external forces and.
3- Stress due to combination of stresses at (1) and (2).

1- Internal stresses due to screwing up forces.

a- Tensile stress due to stretching of the bolt (σ_t).

The initial tension in a bolt, based on experiments , may be found by the relation.

$$F_i = 2840 \times d$$

Where:- F_i – Initial tension in a bolt (N)
d – Nominal diameter of bolt (mm)

The above relation is used for making a joint fluid tight like steam engine cylinder cover joints etc. When the joint is not required as tight as fluid – tight joint, then the initial tension a bolt may be reduced to half of the above value. In such cases.

$$F_i = 1420 \times d$$

The small diameter may fail during tighting, therefore bolts of smaller diameter (less than M 16 or M 18) are not permitted in making fluid tight joints.

If the bolt is not initially stressed, then the maximum safe axial load which may be applied to it, is given by:-

P = Permissible stress × Cross-sectional area at bottom of the thread (i .e. stress area)

The stresses area may be obtained from Table (1) or it may be found by using the relation,

$$Stress\ area = \frac{\pi}{4}\left(\frac{d_p + d_c}{2}\right)^2$$

Where: d_p – Pitch diameter, and
d_c – Core or minor diameter.

$$\sigma_t = \frac{F_i}{A} \qquad \sigma_t = \frac{F_i}{\frac{\pi}{4} d_c^2}$$

b- Torsional shear stress caused by the frictional resistance of the threads during its tightening.

$$\frac{T}{J} = \frac{\tau}{r} \quad \Rightarrow \tau = \frac{T}{J} \times r \quad = \frac{T}{\frac{\pi}{32}(d_c)^4} \times \frac{d_c}{2}$$

Where: τ – Torsional shear stress,
T – Torque applied, and
d_c – Minor or core diameter or the thread.

c- Shear stress across the threads.

The average thread shearing stress for the screw is obtained by using the relation:

$$\tau_s = \frac{F}{\pi \times d_c \times b \times n}$$

Where: b – Width of the thread section at the root

The average thread shearing stress for the nut is:

$$\tau_n = \frac{F}{\pi \times d \times b \times n}$$

Where: d – Major diameter.

d- Compression or crushing stress on threads.

The compression crushing stress between the threads (σ_c) may be obtained by using the relation :

$$\tau_n = \frac{F}{\pi\left(d^2 - (d_c)^2\right) n}$$

Where: d – Major diameter,
d_c – Minor diameter, and
n – Number of threads in engagement.

e- Bending stress.

The bending stress (σ_b) induced in the shank of the bolt is given by:

$$\sigma_b = \frac{x.E}{2l}$$

Where: x – Difference in height between the extreme corners of the nut or head,
l – Length of the shank of the bolt, and
E – Young's modulus for the material of the bolt.

Pre test (2)

Circle the correct answer:-

1- A bolt of (M 24 × 2) means that:-
a- the pitch of the threads is 24 mm and depth is 2 mm.
b- the cross- sectional area of the threads is 24 mm.
c- the normal diameter of bolt is 24 mm and there are two threads per cm.
d- the effective diameter of the bolt is 24 mm and there are two threads per cm

2- When a nut is tightened by placing a washer below it, the bolt will be subjected to:-
a- tensile stress.
b- compressive stress.
c- shear stress.
d- none of these.

3- The eye bolts are used for:-
a- transmission of power.
b- locking devices.
c- lifting and transporting heavy machines.
d- absorbing shocks and vibrations.

Table (1): Design dimensions of screw threads, bolts and nuts (Coarse series)

Designation 1	Pitch(mm) 2	Major or Nominal diameter Nut&Bolt (d=D)mm 3	Effective or pitch diameter Net &bolt (dp) mm 4	Minor or core diameter (dc) mm		Depth of Thread (bolt) mm 7	Stress area mm^2 8
				Bolt 5	Nut 6		
M 1	0.25	1.000	0.838	0.693	0.729	0.153	0.460
M 1.2	0.25	1.200	1.038	0.893	0.929	0.158	0.732
M 1.4	0.3	1.400	1.205	1.032	1.075	0.184	0.983
M 1.6	0.35	1.600	1.373	1.171	1.221	0.215	1.27
M 1.8	0.35	1.800	1.573	1.371	1.421	0.215	1.7
M 2	0.4	2.000	1.740	1.509	1.567	0.245	2.07
M 2.2	0.45	2.200	1.908	1.648	1.713	0.276	2.48
M 2.5	0.45	2.500	2.208	1.948	2.013	0.276	3.39
M 3	0.5	3.000	2.675	2.387	2.459	0.307	5.03
M 3.5	0.6	3.500	3.110	2.764	2.850	0.368	6.78
M 4	0.7	4.000	3.545	3.141	3.242	0.429	8.78
M 4.5	0.75	4.500	4.013	4.580	3.688	0.460	11.3
M 5	0.8	5.000	4.480	4.019	4.134	0.460	14.2
M 6	1	6.000	5.350	4.773	4.918	0.613	20.1
M 7	1	7.000	6.350	5.773	5.918	0.613	28.9
M 8	1.25	8.000	7.188	6.466	6.647	0.767	36.6
M 10	1.5	10.000	9.026	8.160	8.876	0.920	58.3
M 12	1.75	12.000	10.863	9.858	10.106	1.074	84.0
M 14	2	14.000	12.701	11.546	11.835	1.227	115
M 16	2	16.000	14.701	13.546	13.835	1.227	157
M 18	2.5	18.000	16.376	14.933	15.294	1.534	192
M 20	2.5	20.000	18.376	16.933	17.294	1.534	245
M 22	2.5	22.000	20.376	18.933	19.294	1.534	303
M 24	3	24.000	22.051	20.320	20.752	1.840	353
M 27	3	27.000	25.051	23.320	23.752	1.840	459
M 30	3.5	30.000	27.727	25.706	26.211	2.147	561
M 33	3.5	33.000	30.727	28.706	29.211	2.147	694
M 36	4	36.000	33.402	31.093	31.670	2.454	617
M 39	4	39.000	36.402	34.093	34.670	2.454	976
M 42	4.5	42.000	39.077	36.416	37.129	2.760	1104
M 45	4.5	45.000	42.077	39.416	40.129	2.760	1300
M 52	5	52.000	48.752	45.795	46.587	3.067	1755
M 56	5.5	56.000	52.428	49.177	50.046	3.067	2022
M 60	5.5	60.000	56.428	53.177	54.046	3.374	2360

Table(2) : Design dimensions of screw threads, bolts and nuts (Fine series)

Designation	Pitch(mm)	Major or Nominal diameter Nut&Bolt (d=D)mm	Effective or pitch diameter Net &bolt (dp) Mm	Minor or core diameter (dc) mm		Depth of Thread (bolt) mm	Stress area mm^2
				Bolt	Nut		
1	2	3	4	5	6	7	8
M 8 × 1	1	8.000	7.350	6.773	6.918	0.613	39.2
M 10 × 1.25	1.25	10.000	9.188	8.466	8.647	0.767	61.6
M 12 × 1.25	1.25	12.000	11.184	10.466	10.647	0.767	92.1
M 14 × 1.5	1.5	14.000	13.026	12.160	12.376	0.920	125
M 16 × 1.5	1.5	16.000	15.026	14.160	14.376	0.920	167
M 18 × 1.5	1.5	18.000	17.026	16.160	16.376	0.920	216
M 20 × 1.5	1.5	20.000	19.026	18.160	18.376	0.920	272
M 22 × 1.5	1.5	22.000	21.026	20.160	20.376	0.920	333
M 24 × 2	2	24.000	22.701	21.546	21.835	1.227	384
M 27 × 2	2	27.000	25.701	24.546	24.835	1.227	496
M 30 × 2	2	30.000	28.701	27.546	27.835	1.227	621
M 33 × 2	2	33.000	31.701	30.546	30.835	1.227	761
M 36 × 3	3	36.000	34.051	32.319	32.752	1.840	865
M 39 × 3	3	39.000	37.051	35.319	35.752	1.840	1028

Example (1): Determine the safe tensile load for a bolt of (M 30), assuming a safe tensile stress of (42 MPa). Assumed that the bolt is not initially stressed.
Solution:- Given: d = 30 mm, σ_t = 42 MPa = 42 N / mm^2
From table (1), at M 30 $\Longrightarrow$ Stress area = 561 mm^2
$\therefore$ Safe tensile load = Stress × σ_t = 561 × 42 = 23562 N = 23.562 kN

Example (2):Two machine parts are fastened together tightly by means of a (24 mm) tap bolt. If the load tending to separate these parts is neglected, find the stress that is set up in this bolt by the initial tightening.
Solution:- Given: d = 24 mm
From table (1), at M 24 $\Longrightarrow d_c$ = 20.32 mm

$$F_i = 2840d \quad = 2840 \times 24 = 68160N$$

$$\sigma_t = \frac{F_i}{A} \quad = \frac{F_i}{\frac{\pi}{4} d_c^2} \quad = \frac{68160}{\frac{\pi}{4}(20.32)^2} = 210 N/mm^2 \quad \text{or } 210\, MPa$$

2- Stress due to external forces.

The following stresses are induced in a bolt when it is subjected to an external load.

a- Tensile stress.

b- Shear stress.

c- Combined tensile and shear stress.

a- Tensile stress.

The bolts, studs and screws usually carry a load in the direction of the bolt axis which induces a tensile stress in the bolt.

$$\sigma_t = \frac{F}{A} \quad \Rightarrow \quad F = \frac{\pi}{4}(d_c)^2 \times \sigma_t \quad \Rightarrow d_c = \sqrt{\frac{4P}{\pi \sigma_t}}$$

Where: d_c – Root or core diameter of the thread, and

σ_t – Permissible tensile stress for the bolt material.

Now from table (1) the value of the nominal diameter of bolt corresponding to the value of (d_c) may be obtained or stress area $\left[\frac{\pi}{4}(d_c)^2\right]$ may be fixed.

Notes: a- If the external load is taken up by a number of bolts, then

$$F = \frac{\pi}{4}(d_c)^2 \times \sigma_t \times n$$

b- In case the standard table is not available, then for coarse threads,

$d_c = 0.84\ d$, where (d) is the nominal diameter of bolt.

b- Shear stress.

Sometimes, the bolts are use to prevent the relative movement of two or more parts, as in case of flange coupling, then the shear stress is induced in the bolts. The shear stresses should be avoided as far as possible. It should be noted that when the bolts are subjected to direct shearing loads, they should be located in such a way that the shearing load comes upon the body (i.e., shank) of the bolt and not upon the threaded portion. In some cases, the bolts may be relieved of shear load by using shear pins. When a number of bolts are used to share the shearing load, the finished bolts should be fitted to the reamed holes.

Shearing load carried by the bolts, $F_s = \frac{\pi}{4} d^2 \times \tau \times n$ or

$$d_c = \sqrt{\frac{4F}{\pi \tau}}$$

Where: d – Major diameter of the bolt, and
n – Number of bolts.

c- Combined tension and shear stress.

When the bolt is subjected to both tension and shear loads, as in case of coupling bolts or bearing, then the diameter of the shank of the bolt is obtained from the shear load and that of threaded part from the tensile load. A diameter slightly larger, than that required for either shear or tension may be assumed and stresses due to combined load should be checked for the following principal stresses.

Maximum principal shear stress, $\tau_{\max} = \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2}$

And maximum principal tensile stress, $\sigma_{t(\max)} = \frac{\sigma_t}{2} + \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2}$

These stresses should not exceed the safe permissible values of stresses.

Example (3): An eye bolt is to be used for lifting a load of (60 kN). Find the nominal diameter of the bolt, if the tensile stress is not to exceed (100 MPa). Assume coarse threads.

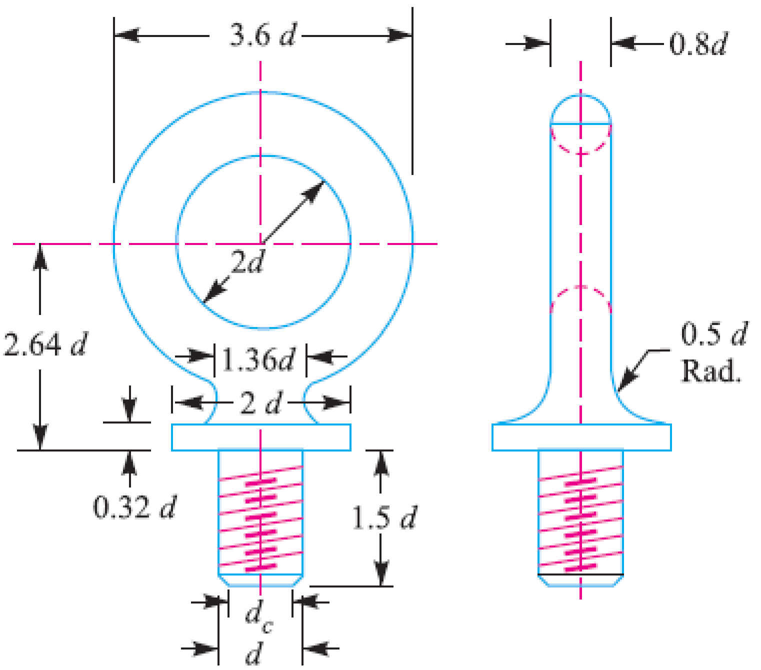
Solution:- Given : F = 60 kN = 60 × 10^3, σ_t = 100 MPa = 100 × 100 N/ mm^2

$$F = \frac{\pi}{4} d_c^2 \times \sigma_t \Rightarrow 60 \times 10^3 = \frac{\pi}{4} (d_c)^2 \times 100 \Rightarrow d_c = 27.6 \ mm$$

From table (1), we find that the standard core diameter (d_c) is (28.706) mm

$\Rightarrow$ nominal diameter (d) is 33 mm

d = 33 mm.



Example (4): Two shafts are connected by means of a flange coupling to transmit torque of (25 kN.mm). The flanges of the coupling are fastened by four bolts of the same material at a radius of (30 mm). Find the size of the bolts if the allowable shear stress for the bolt material is (30 MPa).

Solution:- Given: T = 25 N.m = 25 × 10^3 N.m, n = 4 , R_f = 30 mm,
τ = 30 MPa = 30 N/ mm^2

$$T = F_s \times R_f \Rightarrow F_s = \frac{T}{R_f} = \frac{25 \times 10^3}{30} = 833.33 \ N$$

$\therefore$ Resisting load on the bolts = 833.33 N

$$F_s = \frac{\pi}{4} d_c^2 \times \tau \times n \Rightarrow 833.33 = \frac{\pi}{4} d_c^2 \times 30 \times 4 \Rightarrow d_c = 2.97 mm$$

From table (1) at d_c = 2.97 mm $\Rightarrow$ corresponding size of bolt is M 4

Post test

1- Determine the safe tensile load for bolts of (M20) and (M36). Assume that the bolts are not initially stressed and take the safe tensile stress as (200 MPa).

2- An eye bolt carries a tensile load of (20 kN). Find the size of the bolt, if the tensile stress is not to exceed (100 MPa). Draw a neat proportioned figure for the bolt.

3- An engine cylinder is (300 mm) is diameter and the steam pressure is (0.7 MPa). If the cylinder head is held by 12 studs, find the size, assume safe tensile stress as (280 MPa).

4- Find the size of (14) bolts required for a C. I. steam engine cylinder head. The diameter of the cylinder is (400 mm) and the steam pressure is (0.12 MPa).

Pre test (1)

Question	Answer
1-	b
2-	a
3-	a
4-	d

Pre test (2)

Question	Answer
1-	c
2-	a
3-	c

Post test

Question	Answer
1-	F_1 = 45.038 kN, F_2 = 151.86 kN
2-	M 20
3-	M 24
4-	M 24

References:-

1- A TEXTBOOK OF MACHINE DESIGN – R.S. KHURMI & J.K. GUPTA

وزارة التعليم العالي والبحث العلمي

جامعة الفرات الأوسط التقنية

المعهد التقني – الديوانية

حقيبة رقم (3)

وحده نمطية لدراسة (***Keys***)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة مفهوم الربط بالخابور, أنواعه, استخداماته, حساباته

إعداد

المدرس

فائق حامد جبوري

(3) Keys

3 – 1 Introduction

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

3 – 2 Types of Keys

The following types of keys are important from the subject point of view:
1- Sunk keys, 2- Saddle keys, 3- Tangent keys,
4- Round keys, and 5- Splines.
We shall now discuss the above types of keys, in detail, in the following pages.

3 – 3 Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

1- Rectangular sunk key

A rectangular sunk key is shown in Fig. (3 – 1). The usual proportions of this key are :
Width of key, $\boldsymbol{w = d / 4}$; and thickness of key, $\boldsymbol{t = 2w / 3 = d / 6}$
where d = Diameter of the shaft or diameter of the hole in the hub. The key has taper 1 in 100 on the top side only.

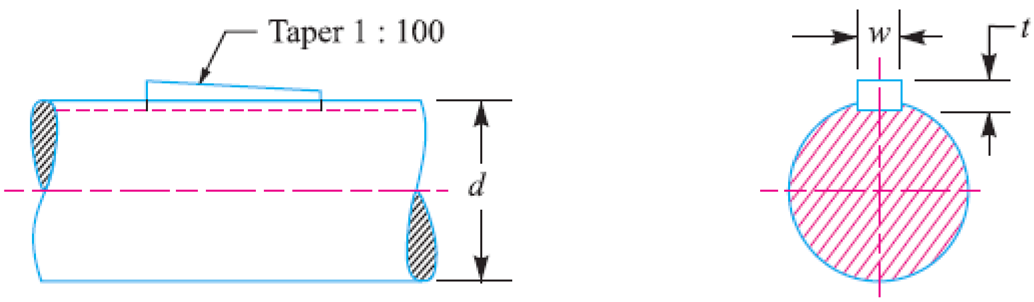


Fig. (3 – 1) Rectangular sunk key.

2- Square sunk key

The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e.

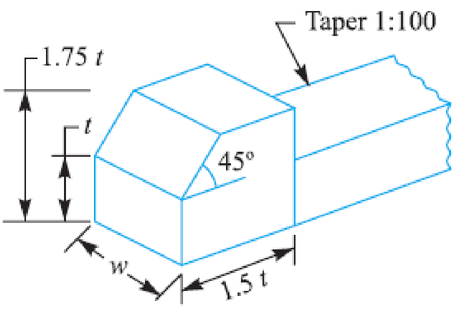
$$w = t = \frac{d}{4}$$

3- Parallel sunk key

The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4- Gib-head key

It is a rectangular sunk key with a head at one end known as **gib head.** It is usually provided to facilitate the removal of key. A gib head key is shown in Fig. (3 - 2 .a) and its use in shown in Fig. (3 - 2 . b).



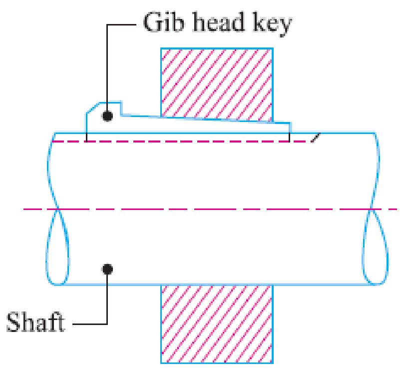


Fig. (3 - 2) Gib-head key.

The usual proportions of the gib head key are :
Width, $\boldsymbol{w = d / 4}$;
and thickness at large end, $\boldsymbol{t = 2w / 3 = d / 6}$

5- Feather key

A key attached to one member of a pair and which permits relative axial movement is known as **feather key.** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

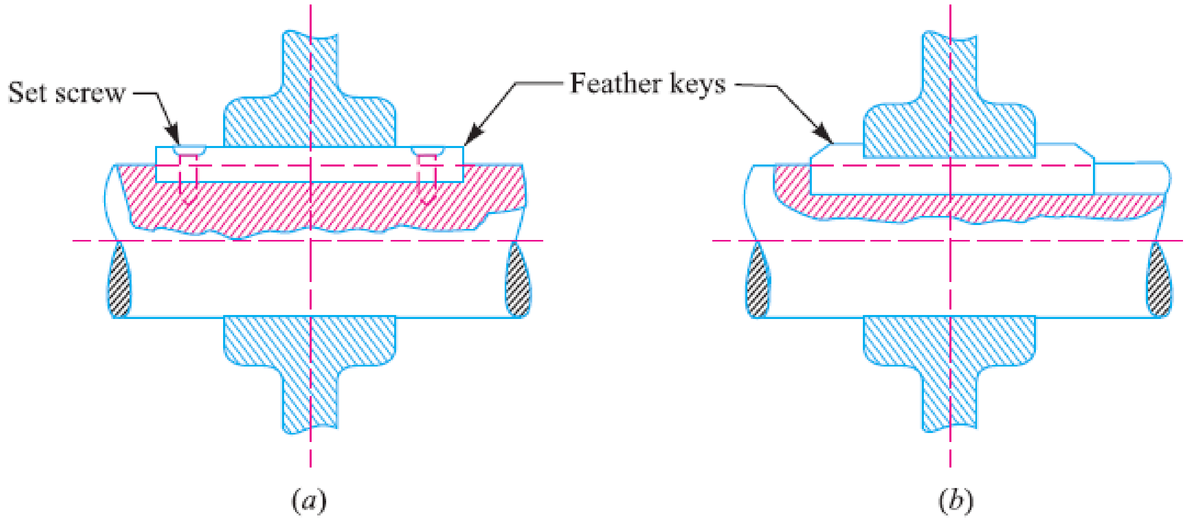


Fig. (3 – 3) Feather key.

The feather key may be screwed to the shaft as shown in Fig. (3 - 3. *a*) or it may have double gib heads as shown in Fig. (3 – 3. *b*). The various proportions of a feather key are same as that of rectangular sunk key and gib head key.

6- Woodruff key

The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. (3 – 4). A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

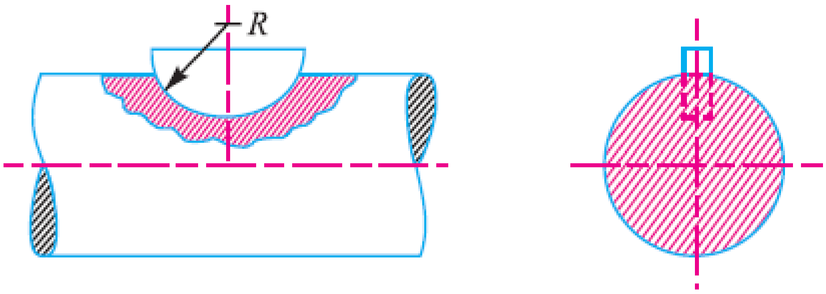


Fig. (3 – 4) Woodruff key

The main advantages of a woodruff key are as follows :

1- It accommodates itself to any taper in the hub or boss of the mating piece.

2- It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turn over in its keyway.

Note : The usual form of rectangular sunk key is very likely to turn over in its keyway unless well fitted as its sides.

The disadvantages are :

1- The depth of the keyway weakens the shaft.

2- It cannot be used as a feather.

7- Saddle keys

The saddle keys are of the following two types:

1- Flat saddle key, and **2-** Hollow saddle key.

A **flat saddle key** is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Fig. (3 – 5). It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

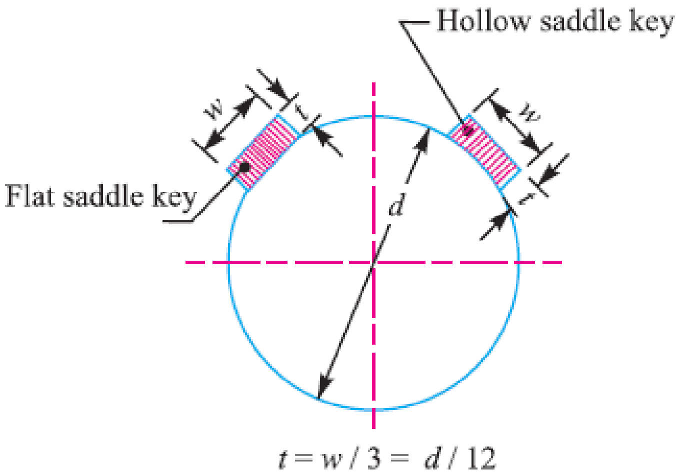


Fig. (3 – 5). Saddle key.

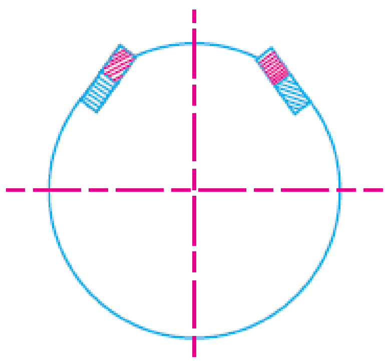


Fig. (3 – 6). Tangent key.

A **hollow saddle key** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

3 – 4 Tangent Keys

The tangent keys are fitted in pair at right angles as shown in Fig. (3 – 6). Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

3 – 6 Round Keys

The round keys, as shown in Fig. (3 – 7.*a*), are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.

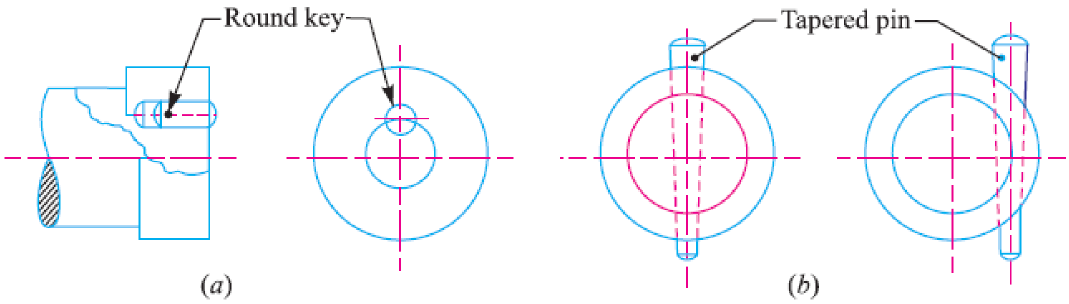


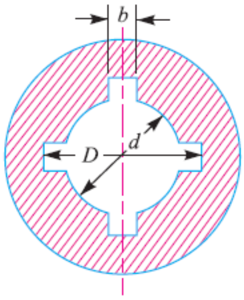
Fig. (3 – 7) Round keys.

Sometimes the tapered pin, as shown in Fig. (3 – 7.*b*), is held in place by the friction between the pin and the reamed tapered holes.

3 - 7 Splines

Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as ***splined shafts*** as shown in Fig. (3 – 8). These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.



$D = 1.25\,d$ and $b = 0.25\,D$ **Fig. (3 – 8). Splines.**

Pre test (1)

1- The taper on a rectangular sunk key is:-

a- 1 in 100 b- 1 in 32 c- 1 in 48 d- 1 in 100

2- The usual proportion for the width of key is:-

a- $\frac{d}{8}$ b- $\frac{d}{6}$ c- $\frac{d}{4}$ d- $\frac{d}{2}$

where (*d*) = Diameter of shaft.

3- When a pulley or other mating piece is required to slide along the shaft, a ----------- sunk key is used:-

a- rectangular b- square c- parallel

4- A key made from a cylindrical disc having segmental cross-section, is known as:-

a- feather key b- gib head key

c- woodruff key d- flat saddle key

3 - 8 Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key:-

1- Forces (F_1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2- Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig. (3 – 9). In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

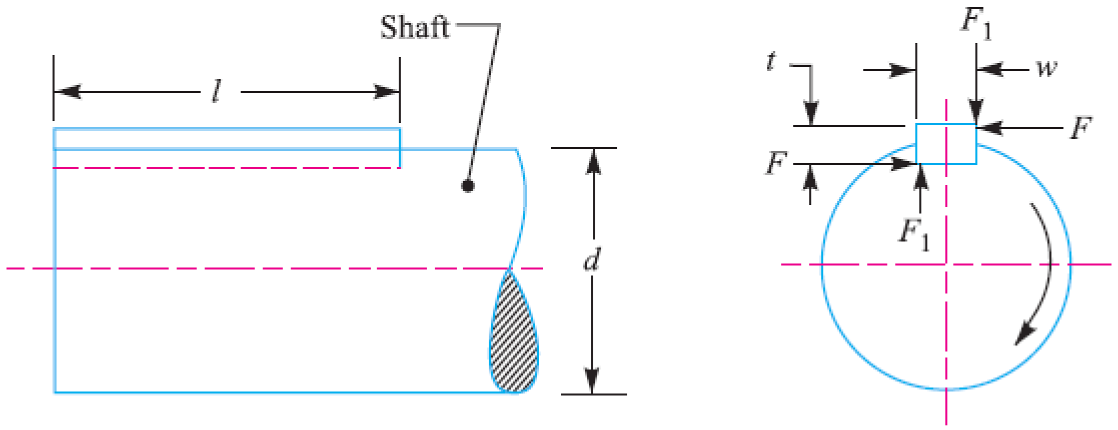


Fig. (3 – 9). Forces acting on a sunk key.

3 - 9 Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig. 13.9.

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key.

t = Thickness of key, and

τ and σ_c = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing. Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

F = Area resisting shearing × Shear stress = $l \times w \times \tau$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2}$$ -------------------- (1)

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

F = Area resisting crushing × Crushing stress $= l \times \frac{t}{2} \times \sigma_c$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$ --------------------- (2)

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$ [Equating equations (1) and (2)]

or $$\frac{w}{t} = \frac{\sigma_c}{2\tau}$$ ---------------------- (3)

and $\tau = \frac{\sigma_c}{2}$ when key is square.

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress. Therefore from equation **(3)**, we have ***w* = *t***. In other words, a square key is equally strong in shearing and crushing. In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2}$$ ---------------------- (4)

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3$$ ---------------------- (5)

...(Taking τ_1 = Shear stress for the shaft material)

From equations (4) and (5), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore\ l = \frac{\pi}{8} \times \frac{\tau_1 \times d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\ d \times \frac{\tau_1}{\tau} \quad \text{(Taking } \boldsymbol{w = d / 4} \text{) ------ (6)}$$

When the key material is same as that of the shaft, then $\tau = \tau_1$

$\therefore\ l = 1.571\ d$ --------------- [From equation (6)]

Pre test (2)

1- The type of stresses developed in the key is/are:-

a- shear stress alone
b- bearing stress alone
c- both shear and bearing stresses
d- shearing, bearing and bending stresses

2- For a square key made of mild steel, the shear and crushing strengths are related as:-

a- shear strength = crushing strength
b- shear strength > crushing strength
c- shear strength
d- none of the above

3- For a key design that requirement is:-

a- (l , w , t)   b- (l , w)   c- (l , t)   d- (l – only)

Example (1): Design the rectangular key for a shaft of (50 mm) diameter, the shearing and crushing stresses in the key are limited to (42 MPa) and (70 MPa). If the width of key is (16 mm) and thickness of the key is (10 mm).
Solution:- Give: d = 50 mm, τ = 42 MPa = 42 N/mm^2, σ_c = 70 MPa = 70 N/mm^2
W = 16 mm , t = 10 mm.

$$T = \frac{\pi}{16} \times \tau \times d^3 \quad \Rightarrow \quad T = \frac{\pi}{16} \times 42 \times (50)^3$$

$$T = 1.03 \times 10^6 \quad N.mm$$

Considering shearing of the key.

$$T = l \times w \times \tau \times \frac{d}{2}$$

$$1.03 \times 10^6 = l \times 16 \times 40 \times \frac{50}{2} \quad \Rightarrow \quad 1.03 \times 10^6 = 16800 \times l$$

$$\therefore l = 61.3 \quad mm$$

Considering crushing of the key.

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \Rightarrow 1.03 \times 10^6 = l \times \frac{10}{2} \times 70 \times \frac{50}{2} \quad \Rightarrow$$

$$1.03 \times 10^6 = 8750 \times l \Rightarrow l = 117.7 \quad mm \qquad \therefore l = 117.7 \quad mm$$ **Ans.**

Example (2): A (15 kW), (960 r.p.m). motor has a mild steel shaft of (40 mm) diameter and the extension being (75 mm). The permissible shear and crushing stresses for the mild steel key are (56 MPa) and (112 MPa). Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.
Solution:- Given: *P* = 15 kW = 15 × 103 W , *N* = 960 r.p.m. , *d* = 40 mm , *l* = 75 mm , τ = 56 MPa = 56 N/mm^2 , σ_c = 112 MPa = 112 N/mm^2.
We know that the torque transmitted by the motor,

$$P = \frac{2\pi NT}{60} \quad \Rightarrow \quad T = \frac{P \times 60}{2\pi N}$$

$$T = \frac{15 \times 10^3 \times 60}{2\pi \times 960} = 149 \;\; N.m \;\; = 149 \times 10^3 \;\; N.mm$$

Considering the key in shearing. We know that the torque transmitted (T),

$$T = l \times w \times \tau \times \frac{d}{2}$$

$$149 \times 10^3 = 75 \times w \times 56 \times \frac{40}{2} \quad \Rightarrow \quad 149 \times 10^3 = 84 \times 10^3 \;\; w$$

$$w = 1.8 \ mm$$

This width of keyway is too small. The width of keyway should be at least d / 4.

$$\therefore \ w = \frac{d}{4} = \frac{40}{4} = 10 \ mm$$ **Ans.**

Since $\boldsymbol{\sigma_c = 2\tau}$, therefore a square key of

w = 10 mm

and t = 10 mm is adopted.

Post test

1- Design a key to transmit (40 kW) power by a shaft of (50 mm) diameter which rotates (1200 r.p.m), if the permissible shearing and crushing stresses are (35 N/mm^2) and (65 N/mm^2) respectively , take the key length (30 mm).

2- A belt pulley is fastened to (80 mm) diameter shaft transmitting (75 kW) at (200 r.p.m) by means of key (22 mm) width and (14 mm) thickness. Determine the length of the key. Take : τ = 40 N/mm^2 , σ_c = 100 N/mm^2.

3- A shaft of (50 mm) diameter is used to transmit power (20 kW)at (200 r.p.m), if the width of key is (10 mm), and the thickness of key is equal to width . Find out the length of the key . Take: τ = 50 N/mm^2 , σ_c = 130 N/mm^2

Key

Pre test (1)

Question	Answer
1-	d
2-	c
3-	c
4-	d

Pre test (2)

Question	Answer
1-	c
2-	a
3-	a

Post test

Question	Answer
1-	12.5 mm , 13.058 mm
2-	127.892 mm
3-	76.394 mm

References:-

1- A TEXTBOOK OF MACHINE DESIGN – R.S. KHURMI & J.K. GUPTA

وزارة التعليم العالي والبحث العلمي

جامعة الفرات الأوسط التقنية

المعهد التقني – الديوانية

حقيبة رقم (4)

وحده نمطية لدراسة (*Riveted Joints*)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة مفهوم الربط بالبرشام, أنواعه, استخداماته, حساباته

إعداد

المدرس

فائق حامد جبوري

(4) Riveted Joints

4 - 1 Introduction

A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called ***shank*** or ***body*** and lower portion of shank is known as ***tail,*** as shown in Fig. (4 – 1). The rivets are used to make permanent fastening between the plates such as in structural work, ship building, bridges, tanks and boiler shells. The riveted joints are widely used for joining light metal.

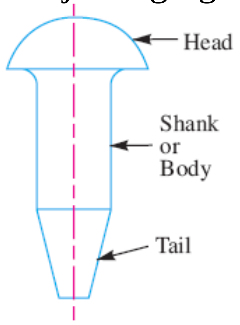


Fig. (4 – 1). Rivet parts.

4 - 2 Methods of Riveting

The function of rivets in a joint is to make a connection that has strength and tightness. The strength is necessary to prevent failure of the joint. The tightness is necessary in order to contribute to strength and to prevent leakage as in a boiler or in a ship hull.

When two plates are to be fastened together by a rivet as shown in Fig. (4 – 2. *a*), the holes in the plates are punched and reamed or drilled. Punching is the cheapest method and is used for relatively thin plates and in structural work. Since punching injures the material around the hole, therefore drilling is used in most pressure-vessel work. In structural and pressure vessel riveting, the diameter of the rivet hole is usually 1.5 mm larger than the nominal diameter of the rivet.

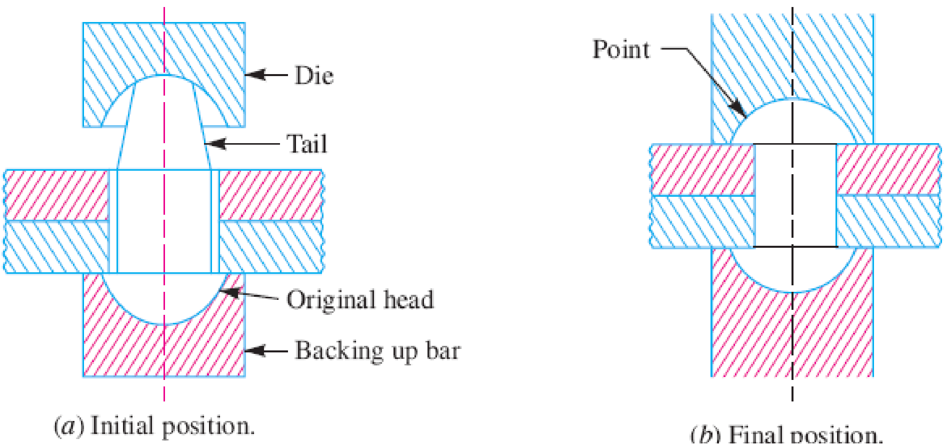


Fig. (4 – 2). Methods of riveting.

Notes:

1- For steel rivets upto (12 mm) diameter, the cold riveting process may be used while for larger diameter rivets, hot riveting process is used.

2- In case of long rivets, only the tail is heated and not the whole shank .

4 - 3 Material of Rivets

The material of the rivets must be tough and ductile. They are usually made of steel (low carbon steel or nickel steel), brass, aluminum or copper, but when strength and a fluid tight joint is the main consideration, then the steel rivets are used.

4 - 6 Types of Rivet Heads

According to Indian standard specifications, the rivet heads are classified into the following three types:

1- Rivet heads for general purposes (below 12 mm diameter) as shown in Fig. (4 – 3).

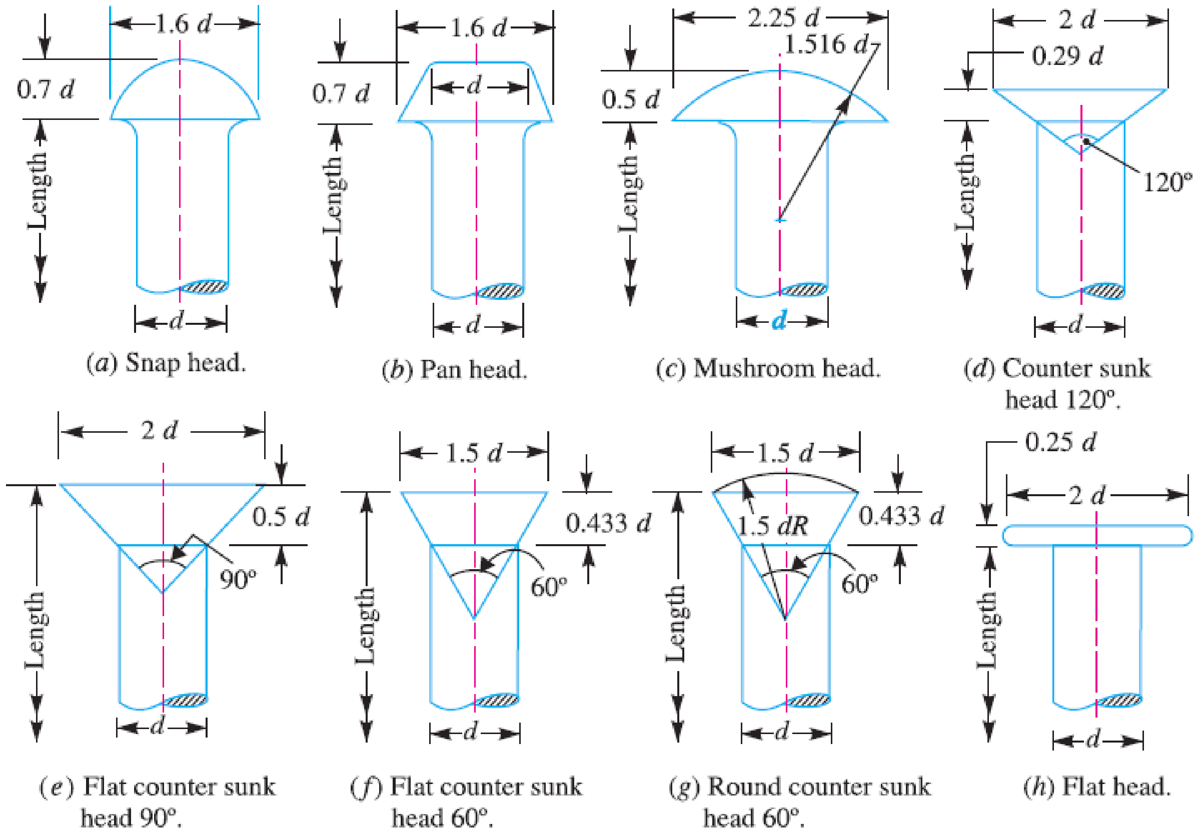


Fig. (4 – 3). Rivet heads for general purposes (below 12 mm diameter).

2- Rivet heads for general purposes (From 12 mm to 48 mm diameter) as shown in Fig. (4 – 4).

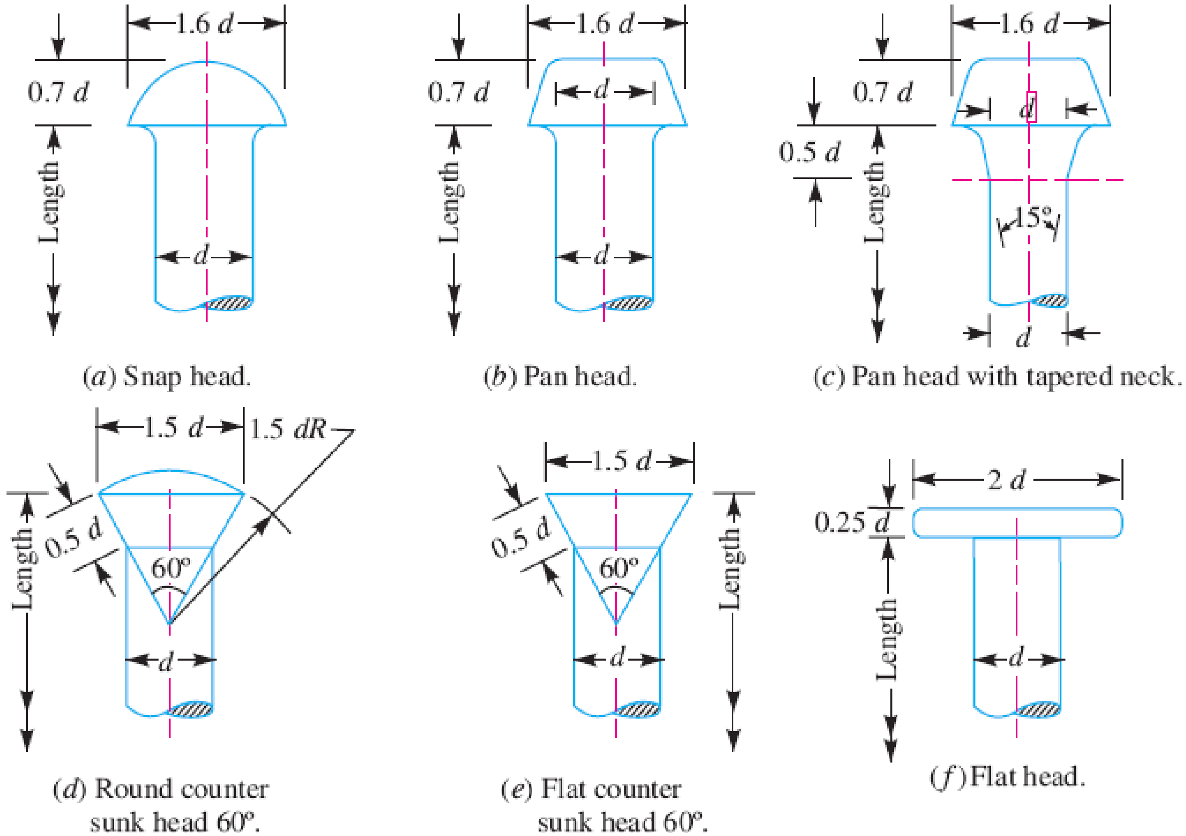


Fig. (4 – 4). Rivet heads for general purposes (from 12 mm to 48 mm diameter)

3- Rivet heads for boiler work (from 12 mm to 48 mm diameter), as shown in Fig. (4-5).

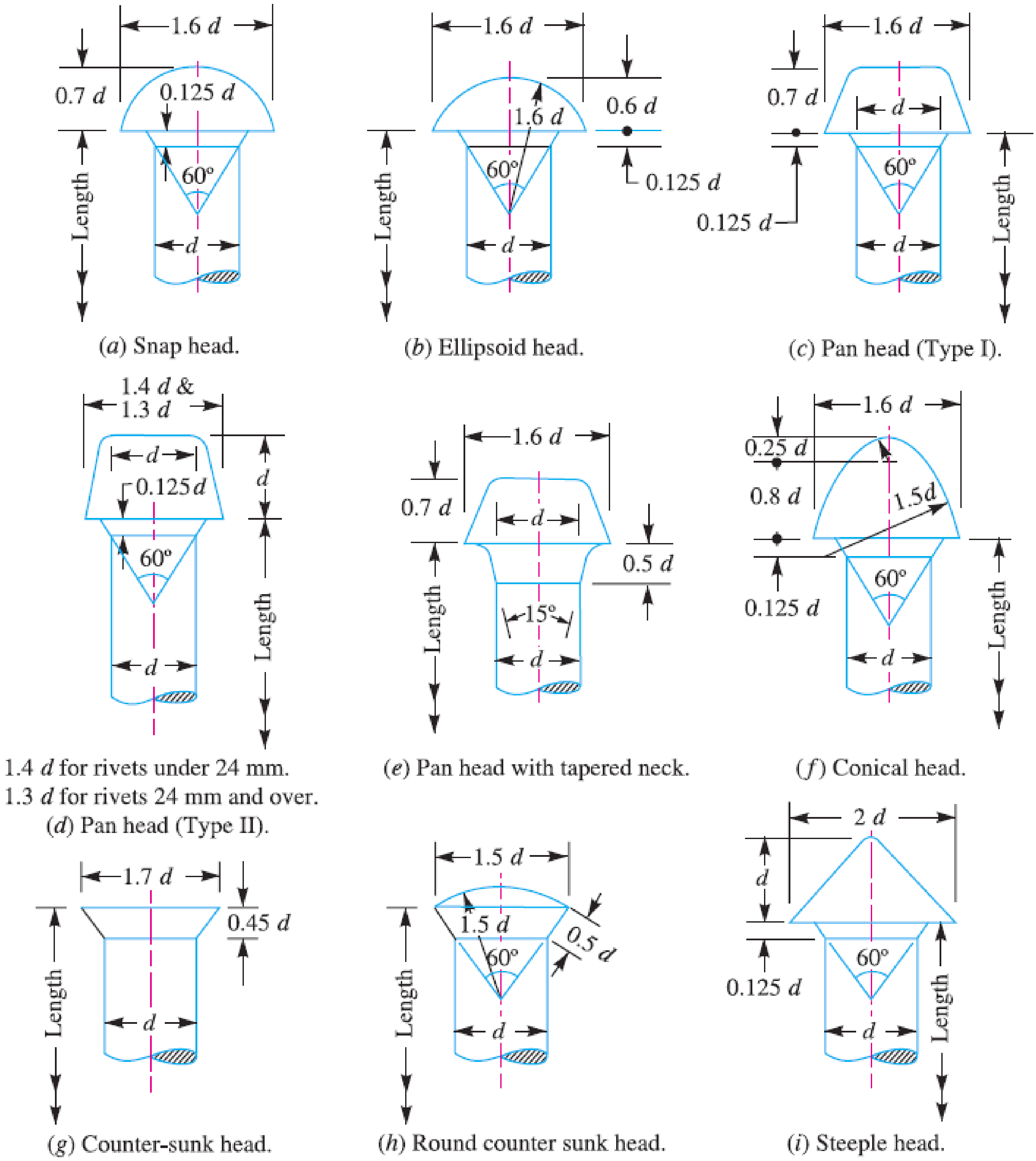


Fig. (4 – 5). Rivet heads for boiler work.

The **snap heads** are usually employed for structural work and machine riveting. The **counter sunk heads** are mainly used for ship building where flush surfaces are necessary. The **conical heads** (also known as **conoidal heads**) are mainly used in case of hand hammering. The **pan heads** have maximum strength, but these are difficult to shape.

4 - 7 Types of Riveted Joints

Following are the two types of riveted joints, depending upon the way in which the plates are connected.

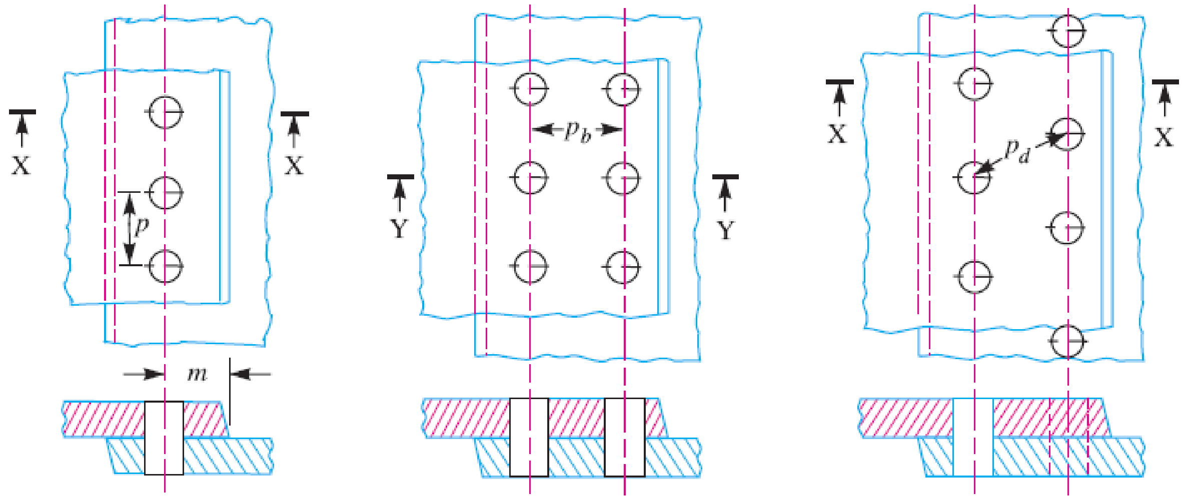
1- Lap joint, and 2- Butt joint.

1- Lap Joint

A lap joint is that in which one plate overlaps the other and the two plates are then riveted together.

1 - Single riveted joint.

2 – Double riveted joint.



(*a*) Single riveted lap joint. (*b*) Double riveted lap joint (Chain riveting). (*c*) Double riveted lap joint (Zig-zag riveting).

Fig. (4 – 6). Single and double riveted lap joints

2- Butt Joint

A butt joint is that in which the main plates are kept in alignment butting (*i.e.* touching) each other and a cover plate (*i.e.* strap) is placed either on one side or on both sides of the main plates. The cover plate is then riveted together with the main plates. Butt joints are of the following two types :

1- Single strap butt joint, and **2-** Double strap butt joint.

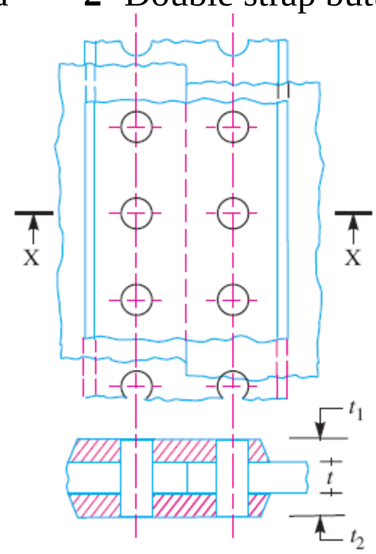


Fig. (4 – 7). Single riveted double strap butt joint.

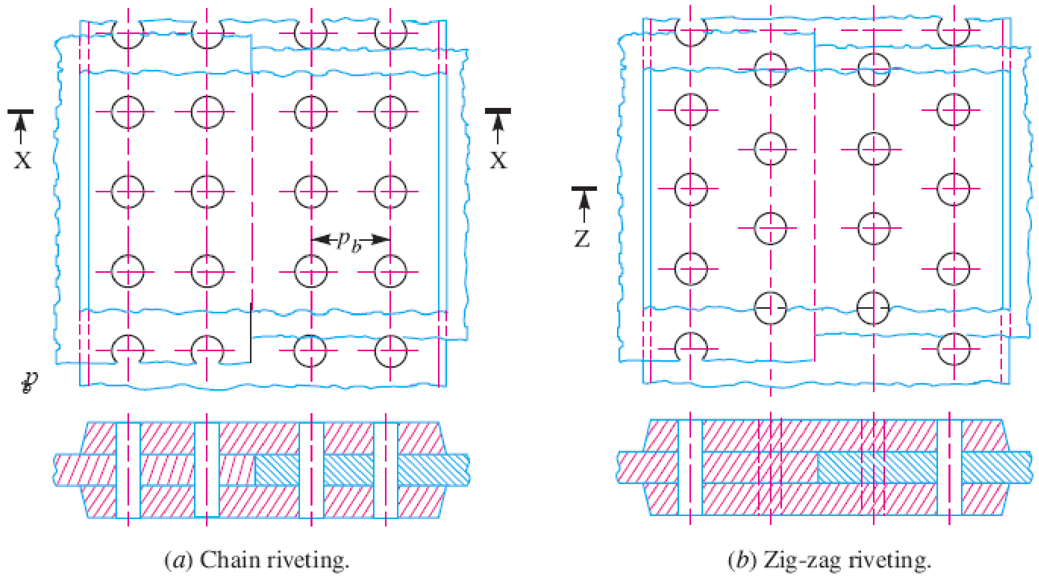


Fig. (4 – 8). Double riveted double strap (equal) butt joints.

4 – 8 Important Terms Used in Riveted Joints

The following terms in connection with the riveted joints are important from the subject point of view:

1- Pitch (*p*):- It is the distance from the centre of one rivet to the centre of the next rivet measured parallel to the seam as shown in Fig. (4 – 6).

2- Back pitch (p_b):- It is the perpendicular distance between the centre lines of the successive rows as shown in Fig. (4 – 6).

3- Diagonal pitch (p_d):- It is the distance between the centres of the rivets in adjacent rows of zig-zag riveted joint as shown in Fig. (4 – 6).

4- Margin or marginal pitch (*m*):- It is the distance between the centre of rivet hole to the nearest edge of the plate as shown in Fig. (4 – 6).

Pre test (1)

Circle the correct answer:-

1- A rivet is specified by:-

a- shank diameter b- length of rivet

c- type of head d- length of tail

2- The diameter of the rivet hole is usually ------------ the nominal diameter of the rivet:-

a- equal to b- less than c- more than

3- A lap joint is always in --------- shear

a- single b- double

4- The rivet head used for boiler plate riveting is usually:-

a- snap head b- pan head

c- counter sunk head d- conical head

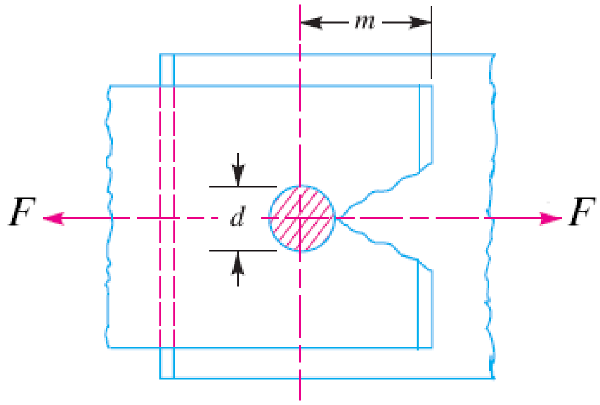
4 – 9 Failures of a Riveted Joint

A riveted joint may fail in the following ways:

1- Tearing of the plate at an edge

A joint may fail due to tearing of the plate at an edge as shown in Fig. (4 – 9). This can be avoided by keeping the margin, $m = 1.5d$

where: d is the diameter of the rivet hole.



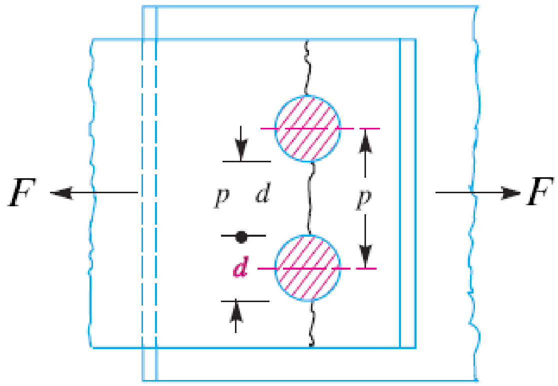


Fig. (4 – 9). Tearing of the plate at an edge. **Fig. (4 – 10). Tearing of the plate across**

The rows of rivets.

2- Tearing of the plate across a row of rivets

Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets as shown in Fig. (4 – 10). In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only.

The resistance offered by the plate against tearing is known as tearing resistance or tearing strength or tearing value of the plate.

Let:- p = Pitch of the rivets,
d = Diameter of the rivet hole,
t = Thickness of the plate, and
σ_t = Permissible tensile stress for the plate material.

We know that tearing area per pitch length,

$$A_t = (p - d) \times t$$

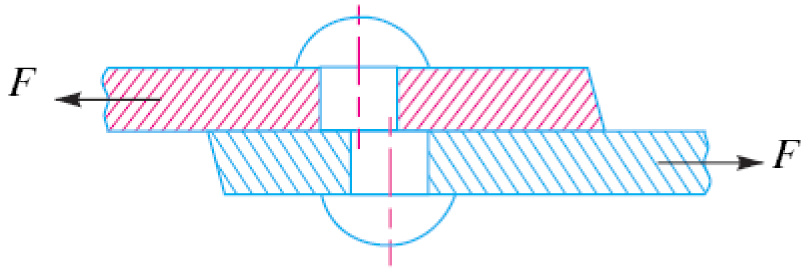
When the tearing resistance (F_t) is greater than the applied load (F) per pitch length, then this type of failure will not occur.

$$F_t = A_t.\sigma_t = (p - d) \times t \times \sigma_t$$

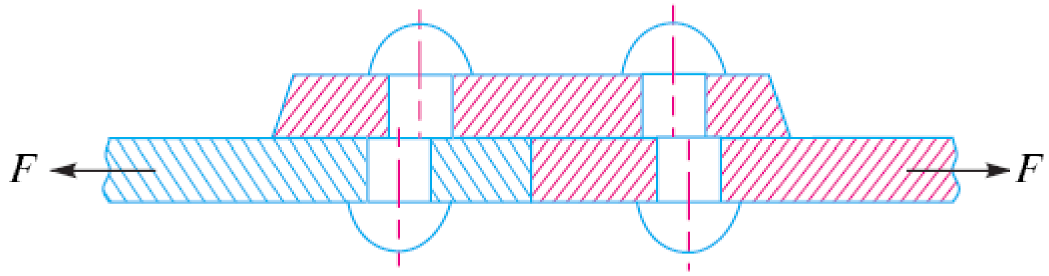
3 - Shearing of the rivets

The plates which are connected by the rivets exert tensile stress on the rivets, and if the rivets are unable to resist the stress, they are sheared off as shown in Fig. (4 – 11).

It may be noted that the rivets are in single shear in a lap joint and in a single cover butt joint, as shown in Fig. (4 – 11). But the rivets are in double shear in a double cover butt joint as shown in Fig. (4 – 12). The resistance offered by a rivet to be sheared off is known as ***shearing resistance*** or ***shearing strength*** or ***shearing value*** of the rivet.



(a) Shearing off a rivet in a lap joint.



(b) Shearing off a rivet in a single cover butt joint.

Fig. (4 – 11). Shearing of rivets.

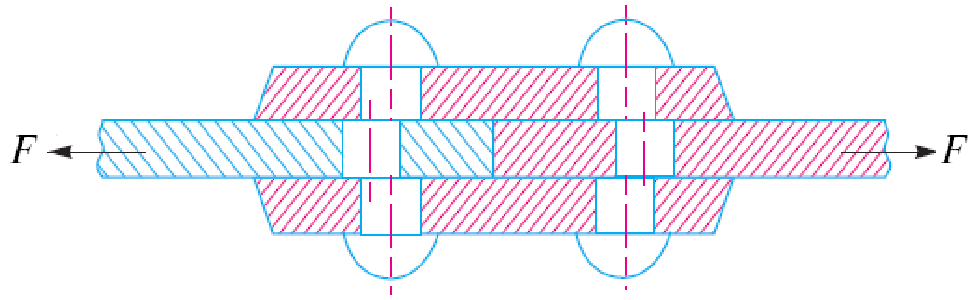


Fig. (4 – 12). Shearing off a rivet in double cover butt joint.

Let d = Diameter of the rivet hole,

τ = Safe permissible shear stress for the rivet material, and

n = Number of rivets per pitch length.

We know that shearing area,

$$A_s = \frac{\pi}{4} \times d^2 \qquad \text{..... (In single shear)}$$

$$A_s = 2 \times \frac{\pi}{4} \times d^2 \qquad \text{...... (Theoretically, in double shear)}$$

$\therefore$ Shearing resistance or pull required to shear off the rivet per pitch length,

$$F_s = n \times \frac{\pi}{4} \times d^2 \times \tau \qquad \text{..... (In single shear)}$$

$$F_s = n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau \qquad \text{..... (in double shear)}$$

4- Crushing of the plate or rivets

Sometimes, the rivets do not actually shear off under the tensile stress, but are crushed as shown in Fig. (4 – 13). Due to this, the rivet hole becomes of an oval shape and hence the joint becomes loose. The failure of rivets in such a manner is also known as **bearing failure.** The area which resists this action is the projected area of the hole or rivet on diametral plane.

The resistance offered by a rivet to be crushed is known as **crushing resistance** or **crushing strength** or **bearing value** of the rivet.

Let d = Diameter of the rivet hole,

t = Thickness of the plate,

σ_c = Safe permissible crushing stress for the rivet or plate material,

and n = Number of rivets per pitch length under crushing.

We know that crushing area per rivet (*i.e.* projected area per rivet),

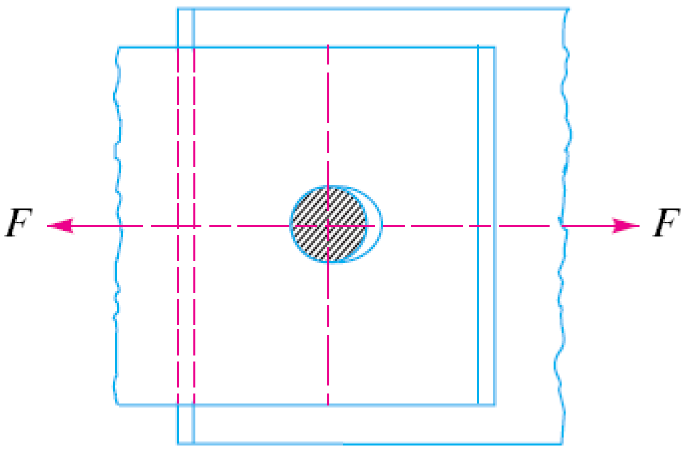


Fig. (4 – 13). Crushing of a rivet.

$$A_c = d\ .\ t$$

$\therefore$ Total crushing area $= n\ .\ d\ .\ t$

and crushing resistance or pull required to crush the rivet per pitch length,

$$F_c = n.d.t \times \sigma_c$$

When the crushing resistance (F_c) is greater than the applied load (F) per pitch length, then this type of failure will occur.

Note : The number of rivets under shear shall be equal to the number of rivets under crushing.

4 - 10 Efficiency of a Riveted Joint (η)

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate.

We have already discussed that strength of the riveted joint
= Least of F_t , F_s and F_c

Strength of the un-riveted or solid plate per pitch length,

$$F = p \times t \times \sigma_t$$

$\therefore$ Efficiency of the riveted joint,

$$\eta = \frac{Least\ of\ F_t, F_s\ and\ F_c}{p \times t \times \sigma_t}$$

Where p = Pitch of the rivets,
t = Thickness of the plate, and
σ_t = Permissible tensile stress of the plate material.

Pre test (2)

1- The strength of the unriveted or solid plate per pitch length is equal to:-

a- $p \times d \times \sigma_t$ b- $p \times t \times \sigma_t$
c- $(p - t)\ d$ d- $(p - d)\ t \times \sigma_t$

2- The centre to centre distance between two consecutive rivets in a row, is called:-

a- margin b- pitch
c- back pitch d- diagonal pitch

Example (1):- Find the efficiency of double lap joint of (6 mm) plates with (2 cm) diameter rivets having a pitch of (6.5 cm), the permissible tensile ,shearing and crushing are (120 MPa), (90 MPa) and (180 MPa) respectively.

Solution:-

1- Tearing resistance of the plate

$$F_t = (p-d) \times t \times \sigma_t$$
$$F_t = (65-20) \times 6 \times 120 = 32400 \ N$$

2- Shearing resistance of the rivets

$$F_s = n \times \frac{\pi}{4} \times d^2 \times \tau$$

$$F_s = 2 \times \frac{\pi}{4} \times (20)^2 \times 90 = 56548.66 \ N$$

3- Crushing resistance of the rivets

$$F_c = n \times d \times t \times \sigma_c$$
$$F_c = 2 \times 20 \times 6 \times 180 = 43200 \ N$$

$\therefore$ Least of F_t , F_s and F_c = 32400 N

Efficiency of the joint $\eta = \dfrac{least\ (F_t\ ,F_s\ ,F_c)}{p\ .t\ \sigma_t}$

$$\Rightarrow \eta = \frac{32400}{65 \times 6 \times 120} = 0.692 \quad or \quad 69.2\%$$

Example (2):- Find the efficiency of a single lap joint of (6 mm) plates with (2 cm) diameter rivets having a pitch of (5 cm), the permissible tensile, shearing and crushing are (120MPa) , (90 MPa) and (180 MPa) respectively .

Solution:-

1- Tearing resistance of the plate

$$F_t = (p-d) \times t \times \sigma_t$$
$$F_t = (50-20) \times 6 \times 120 = 21600 \ N$$

2- Shearing resistance of the rivets

$$F_s = \frac{\pi}{4} \times d^2 \times \tau$$

$$F_s = \frac{\pi}{4} \times (20)^2 \times 90 = 28274.33 \ N$$

3- Crushing resistance of the rivets

$$F_c = d \times t \times \sigma_c$$

$$F_c = 20 \times 6 \times 180 = 21600\ N$$

$\therefore$ Least of F_t , F_s and F_c = 21600 N

Efficiency of the joint

$$\eta = \frac{least\ (F_t\ ,F_s\ ,F_c)}{p\ .t\ \sigma_t}$$

$$\eta = \frac{21600}{50 \times 6 \times 120} = 0.6 \quad or \quad 60\%$$

Example (3):- A double riveted double cover butt joint in plates (20 mm) thick is made with (25 mm) diameter rivets at (b100 mm) pitch. The permissible stresses are : σ_t = 120 MPa, τ = 100 MPa, σ_c = 150 MPa. Find the efficiency of joint, taking the strength of the rivet in double shear as twice than that of single shear.

Solution:-

1- Tearing resistance of the plate

$$F_t = (p - d) \times t \times \sigma_t$$

$$F_t = (100 - 25) \times 20 \times 120 = 180000\ N$$

2- Shearing resistance of the rivets

$$F_s = n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau$$

$$F_s = 2 \times 2 \times \frac{\pi}{4} \times (25)^2 \times 100 = 196375\ N$$

3- Crushing resistance of the rivets

$$F_c = n \times d \times t \times \sigma_c$$

$$F_c = 2 \times 25 \times 20 \times 150 = 150000\ N$$

$\therefore$ Least of F_t , F_s and F_c = 150 000 N

Efficiency of the joint

$$\eta = \frac{least\ (F_t\ ,F_s\ ,F_c)}{p\ .t\ \sigma_t} = \frac{150000}{100 \times 20 \times 120} = 0.624 \quad or \quad 62.5\%$$ **Ans.**

Post test

1- A single riveted lap joint is made in (15 mm) thick plates with (20 mm) diameter rivets. Determine the strength of the joint, if the pitch of rivets is (60 mm). Take σ_t = 120 MPa, τ = 90 MPa and σ_c = 160 MPa.

2- Two plates (16 mm) thick are joined by a double riveted lap joint. The pitch of each row of rivets is (90 mm). The rivets are (25 mm) in diameter. The permissible stresses are as follows :

σ_t = 140 MPa , τ = 110 MPa and σ_c = 240 MPa.

Find the efficiency of the joint.

3- A single riveted double cover butt joint is made in (10 mm) thick plates with (20 mm) diameter rivets with a pitch of (60 mm). Calculate the efficiency of the joint, if

σ_t = 100 MPa ; τ = 80 MPa and σ_c = 160 MPa.

Key

<u>Pre test (1)</u>

Question	Answer
1- 2- 3- 4-	a c a a

<u>Pre test (2)</u>

Question	Answer
1- 2-	b b

<u>Post test</u>

Question	Answer
1- 2- 3-	28280 N 53.5 % 53.8 %

References:-

1- A TEXTBOOK OF MACHINE DESIGN – R.S. KHURMI & J.K. GUPTA

وزارة التعليم العالي والبحث العلمي

جامعة الفرات الأوسط التقنية

المعهد التقني – الديوانية

حقيبة رقم (5)

وحده نمطية لدراسة (*Welded Joints*)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة مفهوم الربط باللحام, أنواعه, استخداماته, حساباته

إعداد

المدرس

فائق حامد جبوري

(5) Welded Joints

5 – 1 Introduction

A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material. The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is extensively used because of greater speed of welding.

The welding processes may be broadly classified into the following two groups:
1- Welding processes that use heat alone e.g. fusion welding.
2- Welding processes that use a combination of heat and pressure e.g. forge welding.

5 – 2 Uses of welded joints

Welding is used in:
1- fabrication as an alternative method for casting or forging .
2- replacement for bolted and riveted joints .
3- repair medium e.g. to reunite metal at a crack .
4- build up a small parts that has broken off such as gear tooth.

5 – 3 Advantages and Disadvantages of Welded Joints over Riveted Joints

Following are the advantages and disadvantages of welded joints over riveted joints.

Advantages
1- The welded structures are usually lighter than riveted structures. This is due to the reason, that in welding, gussets or other connecting components are not used.
2- The welded joints provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.
3- Alterations and additions can be easily made in the existing structures.
4- As the welded structure is smooth in appearance, therefore it looks pleasing.
5- In welded connections, the tension members are not weakened as in the case of riveted joints.
6- A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.
7- Sometimes, the members are of such a shape (*i.e*. circular steel pipes) that they afford difficulty for riveting. But they can be easily welded.
8- The welding provides very rigid joints. This is in line with the modern trend of providing rigid frames.
9- It is possible to weld any part of a structure at any point. But riveting requires enough clearance.

10- The process of welding takes less time than the riveting.

Disadvantages

1- Since there is an uneven heating and cooling during fabrication, therefore the members may get distorted or additional stresses may develop.

2- It requires a highly skilled labour and supervision.

3- Since no provision is kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.

4- The inspection of welding work is more difficult than riveting work.

5 – 4 The main considerations involved in the selection of weld type are:

1- The shape of the welded component required.

2- The thickness of the plates to be welded, and

3- The direction of the forces applied.

5 – 5 Types of Welded Joints

Following two types of welded joints are important from the subject point of view:

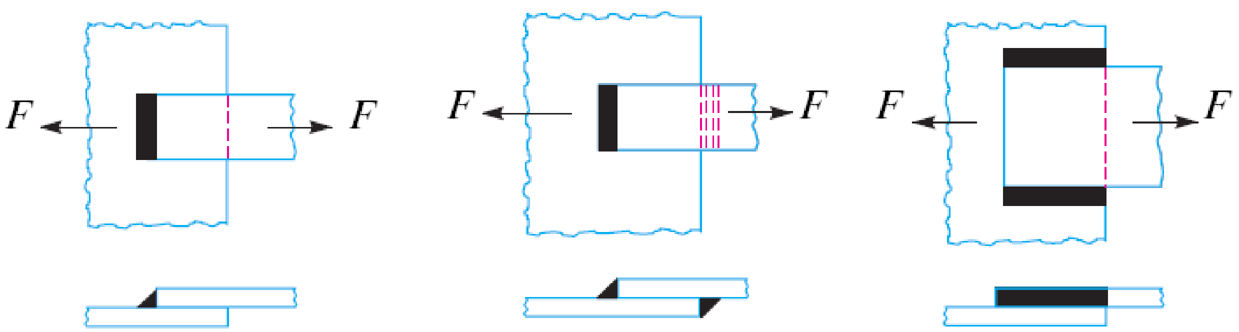
1- Lap joint or fillet joint, and 2- Butt joint.

5 – 6 Lap Joint

The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints may be

1- Single transverse fillet, 2- Double transverse fillet, and 3- Parallel fillet joints.

The fillet joints are shown in Fig. (5 – 1). A single transverse fillet joint has the disadvantage that the edge of the plate which is not welded can buckle or warp out of shape.



(*a*) Single transverse. (*b*) Double transverse. (*c*) Parallel fillet.

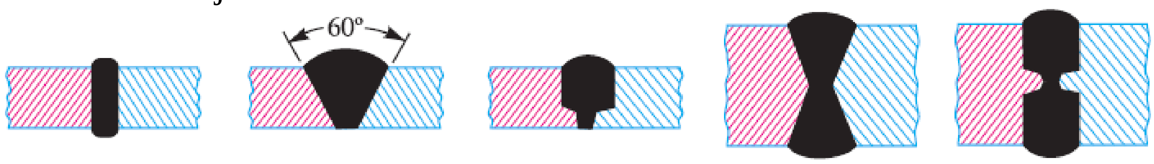
Fig. (5 – 1). Types of lap or fillet joints.

5 – 7 Butt Joint

The butt joint is obtained by placing the plates edge to edge as shown in Fig. (5 – 2). In butt welds, the plate edges do not require beveling if the thickness of plate is less than (5 mm). On the other hand, if the plate thickness is (5 mm to 12.5 mm), the edges should be beveled to V or U-groove on both sides.

The butt joints may be

1- Square butt joint, 2- Single V-butt joint
3- Single U-butt joint, 4- Double V-butt joint, and
5- Double U-butt joint.

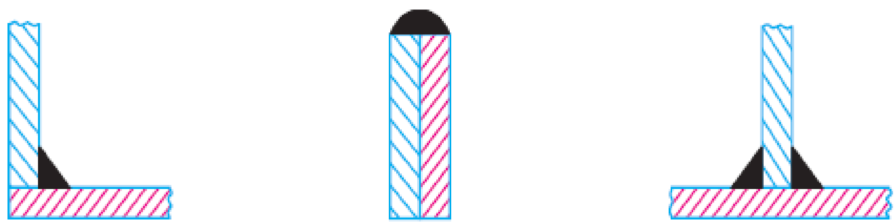


(*a*) Square butt joint. (*b*) Single *V*-butt joint. (*c*) Single *U*-butt joint. (*d*) Double *V*-butt joint. (*e*) Double *U*-butt joint.

Fig. (5 – 2). Types of butt joints.

The other types of welded joints are:

a- corner joint. b- edge joint.
c- T-joint.



(*a*) Corner joint. (*b*) Edge joint. (*c*) *T*-joint.

Fig. (5 – 3). Other types of welded joints.

Pre test (1)

1- In a fusion welding process,

a- only heat is used
b- only pressure is used
c- combination of heat and pressure is used
d- none of these

2- The electric arc welding is a type of ------------ welding.

a- forge b- fusion

5 – 8 Strength of Transverse Fillet Welded Joints

We have already discussed that the fillet or lap joint is obtained by overlapping the plates and then welding the edges of the plates. The transverse fillet welds are designed for tensile strength. Let us consider a single and double transverse fillet welds as shown in Fig. (5 – 4. *a*) and (*b*) respectively.

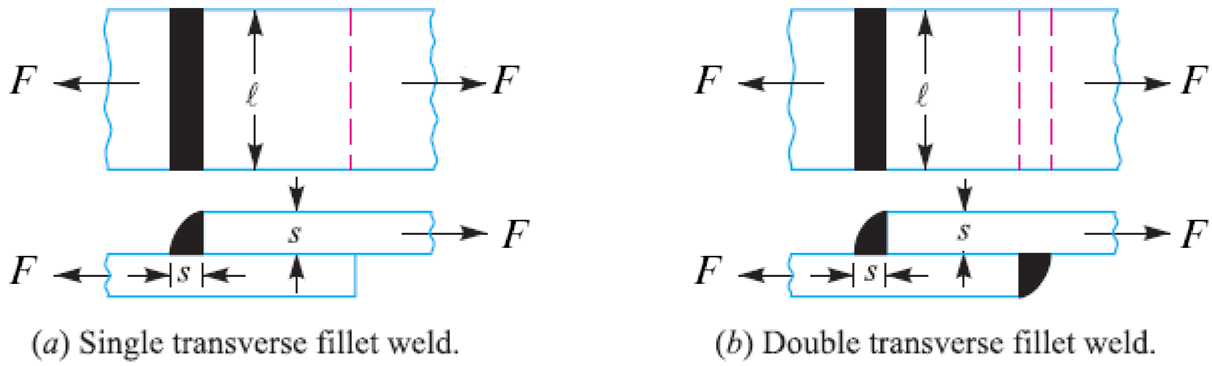


Fig. (5 – 4). Transverse fillet welds.

angled triangle ABC with hypotenuse AC making equal angles with other two sides AB and BC. The enlarged view of the fillet is shown in Fig. (5 – 5). The length of each side is known as **leg** or **size of the weld** and the perpendicular distance of the hypotenuse from the intersection of legs (i.e. BD) is known as **throat thickness.** The minimum area of the weld is obtained at the throat BD, which is given by the product of the throat thickness and length of weld.

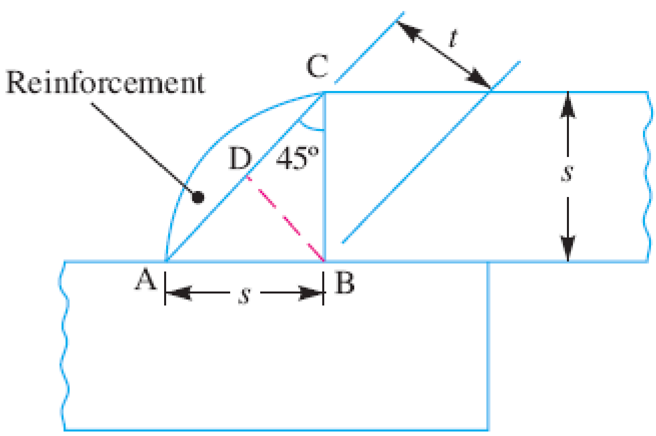


Fig. (5 – 5). Enlarged view of a fillet weld.

Let t = Throat thickness (BD),

s = Leg or size of weld, = Thickness of plate, and

l = Length of weld,

From Fig. (5 – 5), we find that the throat thickness,

$$t = s \times \sin 45° = 0.707\ s$$

∴ Minimum area of the weld or throat area (A)

$$A = \text{Throat thickness} \times \text{Length of weld}$$

$$= t \times l = 0.707\ s \times l$$

If (σ_t)is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,
F = Throat area × Allowable tensile stress

$$F = 0.707 \times s \times l \times \sigma_t$$

and tensile strength of the joint for double fillet weld,

$$F = 2 \times 0.707 \times s \times l \times \sigma_t$$

$$\therefore F = 1.414 \times s \times l \times \sigma_t$$

or $$F = \sqrt{2} \times l \times t \times \sigma_t$$

Note:
1- Since the weld is weaker than the plate due to slag and blow holes, therefore the weld is given a reinforcement which may be taken as 10% of the plate thickness.
2- The minimum area of the weld is taken because the stress is maximum at the minimum area.

5 – 9 Strength of Parallel Fillet Welded Joints

The parallel fillet welded joints are designed for shear strength. Consider a double parallel fillet welded joint as shown in Fig.(5 – 6.*a*). We have already discussed in the previous article, that the minimum area of weld or the throat area,

$$A = 0.707 \times s \times l$$

If (τ) is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,

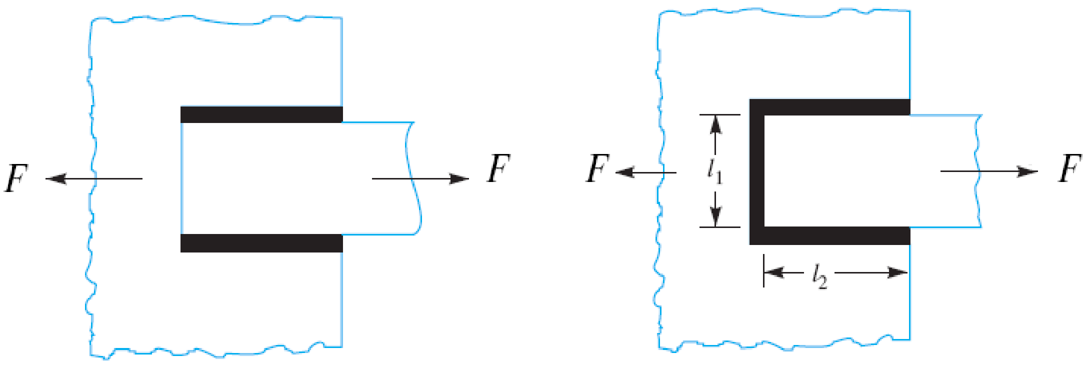
$$F = Throat\ area \times Allowable\ shear stress$$

$$F = 0.707 \times l \times \tau$$

and shear strength of the joint for double parallel fillet weld,

$$F = 2 \times 0.707 \times l \times \tau \Rightarrow F = 1.414 \times l \times \tau$$

or $$F = \sqrt{2} \times l \times t \times \tau$$



(*a*) Double parallel fillet weld.

(*b*) Combination of transverse and parallel fillet weld.

Fig. (5 – 6)

Notes:

1- If there is a combination of single transverse and double parallel fillet welds as shown in Fig. (5 – 6.*b*), then the strength of the joint is given by the sum of strengths of single transverse and double parallel fillet welds. Mathematically,

$$F = 0.707 \times l_1 \times \sigma_t + 1.414 \times l_2 \times \tau$$

Where:- l_1 = is normally the width of the plate.

2- In order to allow for starting and stopping of the bead, (12.5 mm) should be added to the length of each weld obtained by the above expression.

3- For reinforced fillet welds, the throat dimension may be taken as (0.85 t).

5 – 10 Strength of Butt Joints

The butt joints are designed for tension or compression. Consider a single V-butt joint as shown in Fig. (5 – 7.*a*).

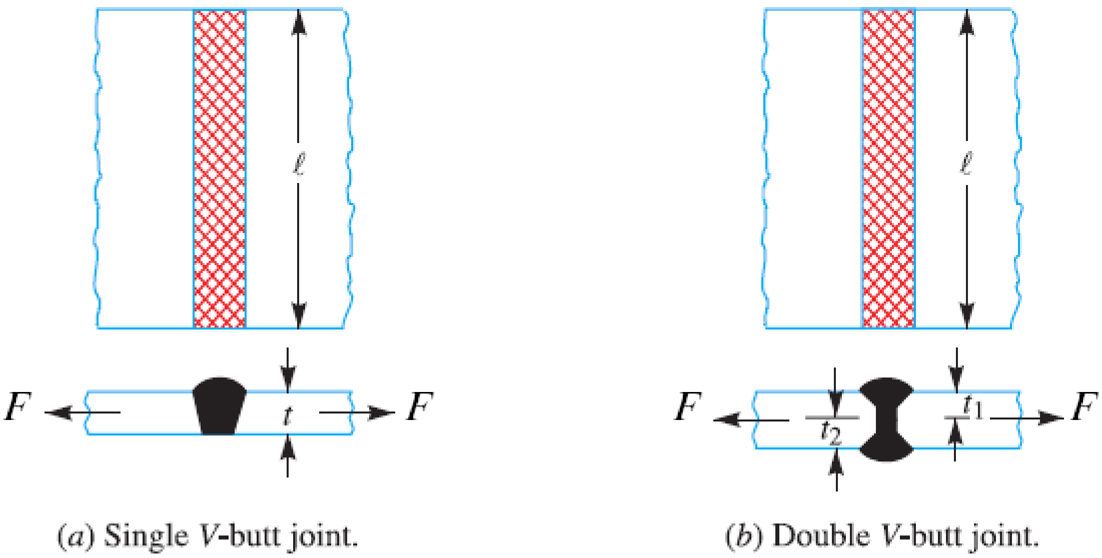


Fig. (5 – 7). Butt joints.

In case of butt joint, the length of leg or size of weld is equal to the throat thickness which is equal to thickness of plates.

∴ Tensile strength of the butt joint (single – *V* or square butt joint),

$$F = t \times l \times \sigma_t$$

Where:- l = Length of weld. It is generally equal to the width of plate.

and tensile strength for double-*V* butt joint as shown in Fig. (5 – 7.*b*) is given by:-

$$F = (t_1 + t_2) \times l \times \sigma_t$$

Where t_1 = Throat thickness at the top, and

t_2 = Throat thickness at the bottom.

5 – 11 Stress Concentration Factor for Welded Joints

The reinforcement provided to the weld produces stress concentration at the junction of the weld and the parent metal. When the parts are subjected to fatigue loading, the stress concentration factor as given in the following table should be taken into account.

Table (5 – 1) Stress concentration factor for welded joints.

Type of joint	Stress concentration factor
1- Reinforced butt welds	1.2
2- Toe of transverse fillet weld	1.5
3- End of parallel fillet weld	2.7
4- T – butt joint with sharp corner	2.0

Note: For static loading and any type of joint, stress concentration factor is 1.0.

Pre test (2)

1- In transverse fillet welded joint, the size of weld is equal to:-

a- $0.5 \times$ Throat of weld b- Throat of weld

c- $\sqrt{2} \times$ Throat of weld d- $2 \times$ Throat of weld

2- The transverse fillet welded joints are designed for:-

a- tensile strength b- compressive strength

c- bending strength d- shear strength

3- The size of the weld in butt welded joint is equal to:-

a- $0.5 \times$ Throat of weld b- Throat of weld

c- $\sqrt{2} \times$ Throat of weld d- $2 \times$ Throat of weld

Example (1):- A plate (100 mm) wide and (12.5 mm) thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of (50 kN). Find the length of the weld so that the maximum shear stress does not exceed (56 MPa). Consider the joint first under static loading and then under fatigue loading.

Solution:-
Given: Width = 100 mm , Thickness = 12.5 mm , F = 50 kN = 50 × 10^3 N
τ = 56 MPa = 56 N/mm^2

1- For static load.

$$F = \sqrt{2} \times l \times t \times \tau$$

$$l = \frac{F}{\sqrt{2} \times t \times \tau} = \frac{50 \times 10^3}{\sqrt{2} \times 12.5 \times 56} = 50.5 \ mm$$

$$\therefore l = 50.5 + 12.5 = 63 \ mm$$

2- For fatigue loading.

$\Rightarrow$ s = 2.7 parallel fillet weld (stress concentration factor)

$$\tau_d = \frac{\tau}{s} = \frac{56}{2.7} = 20.74 \ MPa$$

$$l = \frac{F}{\sqrt{2} \times t \times \tau_d} = \frac{50 \times 10^3}{\sqrt{2} \times 12.5 \times 20.74} = 136.37 \ mm$$

$$\therefore l = 136.37 + 12.5 = 148.87 \ mm$$

Example (2):- Two steel plates (100 mm) wide and (12.5) thick are to be joined by double transverse fillet weld , the maximum tensile stress is not exceed (70 MPa). Find the length of the weld for:-
1- Static loading , and 2- Dynamic loading.
Solution:-
Given: Width = 100 mm , Thickness = 12.5 mm , σ_t = 70 MPa.
1- For static load.

$$\sigma_t = \frac{F_t}{A} \Rightarrow F_t = \sigma_t \times A \Rightarrow F_t = 70 \times (100 \times 12.5) = 87500 \ N$$

$$F_t = \sqrt{2} \times l \times t \times \sigma_t \Rightarrow 87500 = \sqrt{2} \times l \times 12.5 \times 70$$

$$l = 70.7 \ mm$$

$$\therefore \ l = 70.7 + 12.5 = 83.2 \ mm$$

2- For dynamic load.

$\Rightarrow$ s = 1.5 transverse fillet weld (stress concentration factor)

$$\sigma_d = \frac{\sigma_t}{s} = \frac{70}{1.5} = 46.667 \ MPa$$

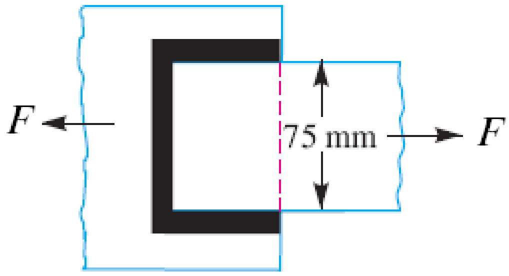
$$F_t = \sqrt{2} \times l \times t \times \sigma_d \implies 87500 = \sqrt{2} \times l \times 12.5 \times 46.667$$

$$l = 106 \ mm$$

$$\therefore \ l = 106 + 12.5 = 118.5 \ mm$$

Post test

1- A plate (75 mm) wide and (12.5 mm) thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. below. The maximum tensile and shear stresses are (70 MPa) and (56 MPa) respectively. Find the length of each parallel fillet weld, if the joint is subjected to both static and fatigue loading.



Key

<u>Pre test (1)</u>

Question	Answer
1-	a
2-	b

<u>Pre test (2)</u>

Question	Answer
1-	c
2-	a
3-	b

<u>Post test</u>

Question	Answer
1-	62.5 mm , 39.7 mm , 121.3 mm

References:-

1- A TEXTBOOK OF MACHINE DESIGN – R.S. KHURMI & J.K. GUPTA

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هيئة التعليم التقني

المعهد التقني – الديوانية

حقيبة رقم (6)

وحده نمطية لدراسة (SPRINGS)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة مفهوم النوابض, أنواعها, حسابات النوابض اللولبية

إعداد

المدرس

فائق حامد جبوري

(6) Springs

6 – 1 Introduction

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed.

6 – 2 The various important applications of springs are as follows:

1- To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.
2- To apply forces, as in brakes, clutches and spring loaded valves.
3- To control motion by maintaining contact between two elements as in cams and followers.
4- To measure forces, as in spring balances and engine indicators.
5- To store energy, as in watches, toys, etc.

6 – 3 Types of Springs

1- Helical springs

The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular.

(*a*) Compression helical spring. (*b*) Tension helical spring.

Fig. (6 – 1). Helical springs.

2- Conical and volute springs

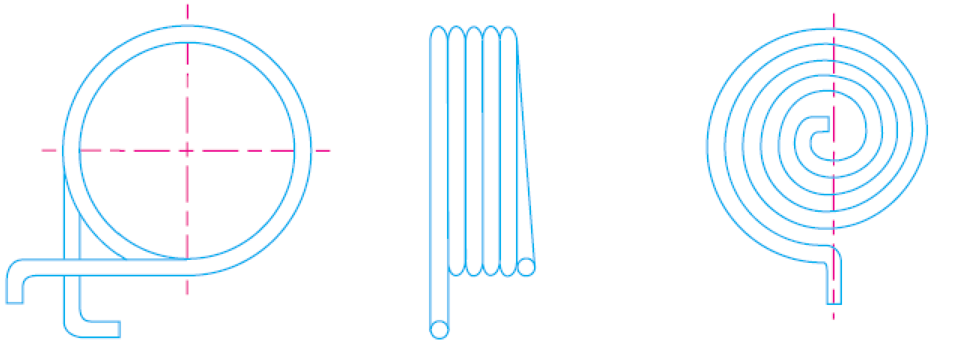
The conical and volute springs, as shown in Fig. (6 – 2), are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired.

(*a*) Conical spring. (*b*) Volute spring.

Fig. (6 – 2). Conical and volute springs.

3- Torsion springs

These springs may be of helical or spiral type as shown in Fig. (6 – 3). The helical type may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The spiral type is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks.



(*a*) Helical torsion spring. (*b*) Spiral torsion spring.

Fig. (6 – 3). Torsion springs.

4- Laminated or leaf springs

The laminated or leaf spring (also known as flat spring or carriage spring) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in Fig. (6 – 4). These are mostly used in automobiles. The major stresses produced in leaf springs are tensile and compressive stresses.

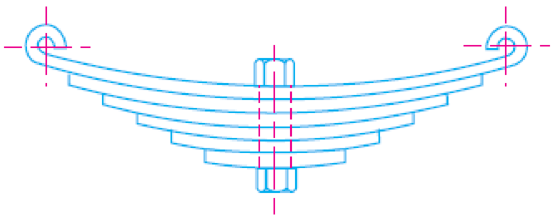


Fig. (6 – 4). Laminated or leaf springs.

5- Disc springs

These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig. (6 – 5). These springs are used in applications where high spring rates and compact spring units are required.

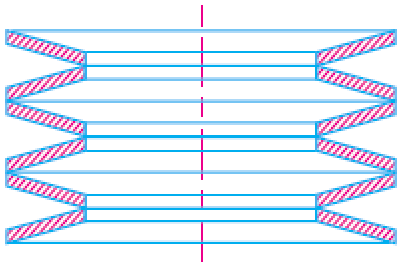


Fig. (6 – 5). Disc springs.

6- Special purpose springs

These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

Pre test (1)

1- A spring used to absorb shocks and vibrations is:-

a- closely-coiled helical spring b- open-coiled helical spring
c- conical spring d- torsion spring

2- The spring mostly used in gramophones is:-

a- helical spring b- conical spring
c- laminated spring d- flat spiral spring

3- Which of the following spring is used in a mechanical wrist watch?

a- Helical compression spring b- Spiral spring
c- Torsion spring d- Bellevile spring

6 – 4 Terms used in Compression Springs

The following terms used in connection with compression springs are important from the subject point of view.

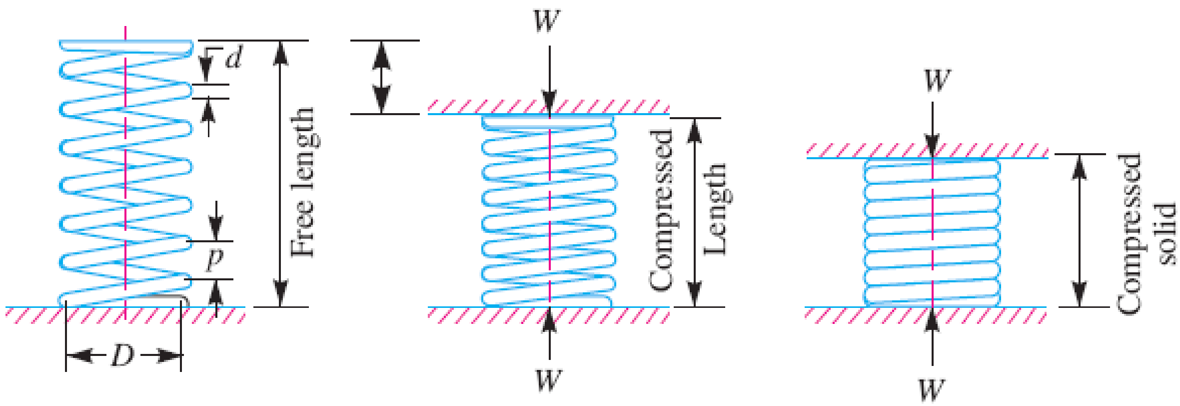


Fig. (6 – 6). Compression spring nomenclature.

1- Solid length (L_S)

When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be **solid.**

$$L_S = n' \times d$$

Where n' = Total number of coils, and
d = Diameter of the wire.

2- Free length (L_F)

The free length of a compression spring, as shown in Fig. (6 – 6), is the length of the spring in the free or unloaded condition.

Free length of the spring,
L_F = Solid length + Maximum compression + Clearance between adjacent coils (or clash allowance)

$$L_F = n' \times d + \delta_{\max} + 0.15\delta_{\max}$$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n' \times d + \delta_{\max} + (n' - 1) \times 1 \ mm$$

In this expression, the clearance between the two adjacent coils is taken as(1mm).

3- Spring index (C)

The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index (C), $$C = \frac{D}{d}$$

Where:- D = Mean diameter of the coil, and
d = Diameter of the wire.

4- Spring rate (*k*)

The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate *(k)*, $$k = \frac{W}{\delta}$$

Where:- W = Load, and
δ = Deflection of the spring.

Note:- In actual practice, the compression springs are seldom designed to close up under the maximum working load and for this purpose a clearance (or clash allowance) is provided between the adjacent coils to prevent closing of the coils during service. It may be taken as (15 percent) of the maximum deflection.

5- Pitch (*p*)

The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil *(p)*, $$p = \frac{Free\ length}{n' - 1}$$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

$$p = \frac{L_F - L_S}{n'}$$

Where:- L_F = Free length of the spring,
L_S = Solid length of the spring,
n' = Total number of coils, and
d = Diameter of the wire.

Note:

1- The total number of turns of a tension helical spring must be equal to the number of turns (n) between the points where the loops start plus the equivalent turns for the loops. It has been found experimentally that half turn should be added for each loop. Thus for a spring having loops on both ends, the total number of active turns,

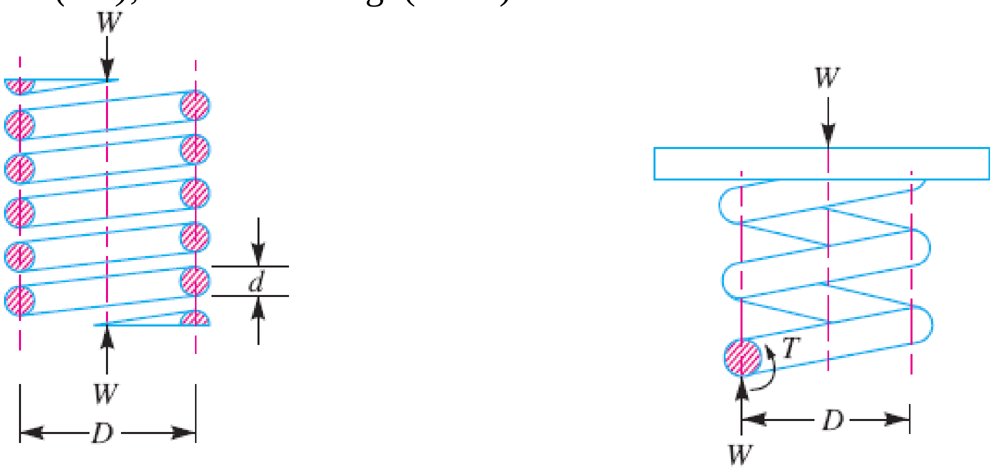
$$n' = n + 1$$

2- (L_F) is more than four times the mean or diameter (D), then the spring under the curvature.

$$L_F > 4D \qquad \text{spring is expose to curvature}$$

6 – 5 Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load *(W)*, as shown in Fig. (6 – 7).



(*a*) Axially loaded helical spring.

(*b*) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Fig. (6 – 7)

1- Torsional shear stress is induced in the wire *(τ_t)*.

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \tau_t = \frac{8.W.D}{\pi d^3} \qquad \text{-------------- (1)}$$

2- Direct shear stress due to the load (τ_d).

$$\tau_d = \frac{Load}{Cross-sectional\ area\ of\ the\ wire}$$

$$\tau_d = \frac{W}{\frac{\pi \times d}{4}} = \frac{4W}{\pi d^2} \qquad \text{-------------- (2)}$$

3- Maximum shear stress induced in the wire *(τ)* .

τ = Torsional shear stress (τ_t) + Direct shear stress (τ_d)

$$\tau = \tau_t + \tau_d = \frac{8.W.D}{\pi d^3} + \frac{4W}{\pi d^3}$$

$$\therefore \ \tau = \frac{8.W.D}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$\Rightarrow \tau = K_S \frac{8.W.D}{\pi d^3} \qquad \text{-------------- (3)}$$

Where:- K_S = Shear stress factor.

$$K_S = 1 + \frac{1}{2C}$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

Where:- $K =$ Wahl's correction factor.

$\therefore$ Maximum shear stress (τ) .

$\tau = K_S \dfrac{8.W.D}{\pi d^3}$ when neglecting the curvature effect.

$\tau = K \dfrac{8.W.D}{\pi d^3}$ when considering the curvature effect.

$\because C = \dfrac{D}{d}$ $\qquad \Rightarrow \tau = K \dfrac{8.W.C}{\pi d^2}$

The values of (K) for a given spring index (C) may be obtained from the graph as shown in Fig. (6 – 8).

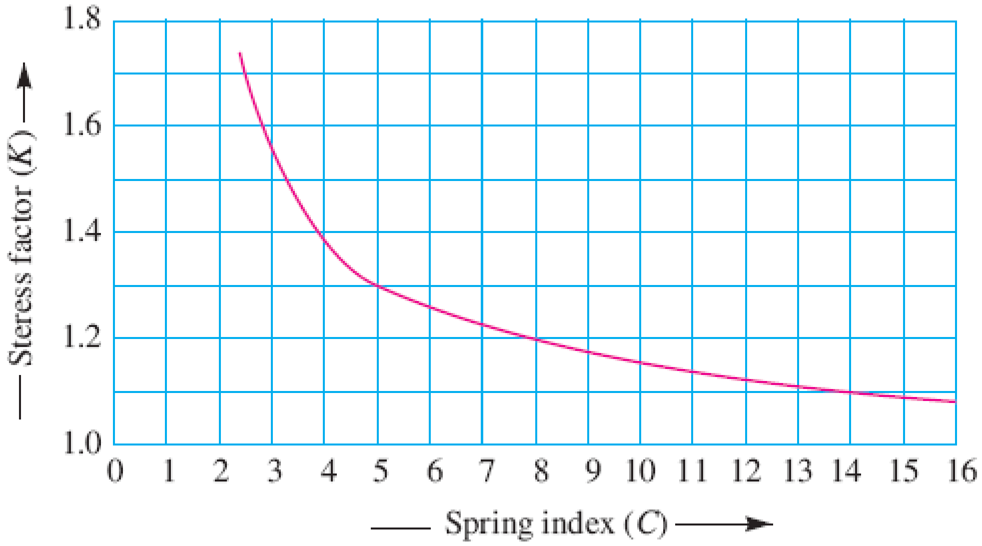


Fig. (6 – 8). Wahl's stress factor for helical springs.

We see from Fig. 23.12 that Wahl's stress factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.

Note:

The Wahl's stress factor (K) may be considered as composed of two sub-factors, (KS) and (KC), such that

$$K = K_S \times K_C$$

Where:- K_S = Stress factor due to shear, and
K_C = Stress concentration factor due to curvature.

6 – 6 Deflection of Helical Springs of Circular Wire

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that

Total active length of the wire,

l = Length of one coil × No. of active coils $= \pi D \times n$

Let θ = Angular deflection of the wire when acted upon by the torque *(T)*.

∴ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad \text{---------- (1)}$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \theta = \frac{T.\,l}{J.G} \qquad [\text{ considering } = \frac{T}{J} = \frac{G.\theta}{l}\ ]$$

Where:- J = Polar moment of inertia of the spring wire.

$$J = \frac{\pi}{32} \times d^4$$

d = the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of *(l)* and *(J)* in the above equation, we have

$$\theta = \frac{T.\,l}{J.G} = \frac{\left(W \times \frac{D}{2}\right) \times \pi.D.n}{\frac{\pi}{32} \times d^4 G}$$

$$\therefore \theta = \frac{16W.D^2.n}{G.d^4} \qquad \text{------------------- (2)}$$

Substituting this value of *(θ)* in equation **(*1*)**, we have

$$\delta = \frac{16W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\ W.D^3.n}{Gd^4} \qquad \because C = \frac{D}{d}$$

$$\therefore \delta = \frac{8.W.C^3.n}{G.d}$$

and the stiffness of the spring or spring rate (k),

$$k = \frac{W}{\delta} \quad \Rightarrow k = \frac{G.d^4}{8.D^3.n} \qquad \therefore k = \frac{G.d}{8.C^3.n}$$

Pre test (2)

1- When a helical compression spring is subjected to an axial compressive load, the stress induced in the wire is:-

a- tensile stress b- compressive stress

c- shear stress d- bending stress

2- In a close coiled helical spring, the spring index is given by (*D*/*d*)where *D* and *d* are the mean coil diameter and wire diameter respectively. For considering the effect of curvature, the Wahl's stress factor *K* is given by:-

a- $\frac{4C-1}{4C+4}+\frac{0.615}{C}$ b- $\frac{4C-1}{4C-4}+\frac{0.615}{C}$

c- $\frac{4C+1}{4C-4}+\frac{0.615}{C}$ d- $\frac{4C+1}{4C-4}+\frac{0.615}{C}$

3- When helical compression spring is cut into halves, the stiffness of the resulting spring will be:-

a- same b- double

c- one-half d- one-fourth

Example (1):- A compression coil spring made of an alloy steel is having the following specifications:
Mean diameter of coil = 50 mm , Wire diameter = 5 mm , Number of active coils = 20. If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.

Solution:- Given : D = 50 mm ; d = 5 mm ; n = 20 ; W = 500 N

We know that the spring index,

$$C=\frac{D}{d}=\frac{50}{5}=10$$

$\therefore$ Shear stress factor,

$$K_S=1+\frac{1}{2C}=1+\frac{1}{2\times 10}=1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\tau=K_S\times\frac{8.W.D}{\pi d^3}=1.05\times\frac{8\times 500\times 50}{\pi\times 5^3}=534.76\ \ N/mm^2$$

$$=534.76\ \ MPa$$ **Ans.**

Example (2):- A helical spring is made from a wire of (6 mm) diameter and has outside diameter of (75 mm). If the permissible shear stress is (350 MPa) and modulus of rigidity (84 kN/mm^2), find the axial load which the spring can carry and the deflection per active turn.

Solution:- Given: d = 6 mm , D_o = 75 mm , τ = 350 MPa = 350 N/mm^2 , G = 84 kN/mm^2 = 84 × 103 N/mm^2

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \ mm$$

$\therefore$ Spring index, $C = \dfrac{D}{d} = \dfrac{69}{6} = 11.5$

1- Neglecting the effect of curvature

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$\tau = K_S \times \frac{8.W.D}{\pi d^3}$$

$$350 = 1.043 \times \frac{8 \times W \times 69}{\pi \times 6^3}$$

$$\Rightarrow W = 412.5 \ N$$ **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8.W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8.W.D^3.}{G.d^4} = \frac{8 \times 412.5 \times (69)^3}{84 \times 10^3 \times (6)^4}$$

$$\therefore \frac{\delta}{n} = 9.96 \ mm$$ **Ans.**

2- Considering the effect of curvature

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

$$K = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$\tau = K \times \frac{8.W.C}{\pi d^2}$$

$$\Rightarrow 350 = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} \qquad \Rightarrow W = 382.4 \ N$$ **Ans.**

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8.W.D^3.}{G.d^4} = \frac{8 \times 383.4 \times (69)^3}{84 \times 10^3 \times (6)^4}$$

$$\therefore \frac{\delta}{n} = 9.26 \ mm$$ **Ans.**

Example (3):- Design a helical compression spring for a maximum load of (1000 N) for a deflection of (25 mm) using the value of spring index as (5). The maximum permissible shear stress for spring wire is (420 MPa) and modulus of rigidity is (84 kN/mm^2).

Take Wahl's factor $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$, where C = Spring index.

Solution:-

Given: W = 1000 N , δ = 25 mm , $C = D/d = 5$, τ = 420 MPa = 420 N/mm^2 , G = 84 kN/mm^2 = 84 × 10^3 N/mm^2

1. diameter of the spring wire.

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5}$$

$$\therefore K = 1.31$$

$$\tau = K \times \frac{8.W.C}{\pi d^2} \quad \Rightarrow \quad 420 = 1.31 \times \frac{8 \times 1000 \times 5}{\pi \times d^2}$$

$$\therefore d = 6.3 \ mm$$

2- Mean diameter of the spring coil.

$$C = \frac{D}{d} \quad \Rightarrow D = C \times d = 5 \times 6.3 = 31.5 \ mm$$

3- Number of turns of the coils (n').

n = Number of active turns of the coils.

$$\delta = \frac{8.W.C^3.n}{G.d} \quad \Rightarrow 25 = \frac{8 \times 1000 \times (5)^3 \times n}{84 \times 10^3 \times 6.3}$$

$\therefore n = 13.23$ say 14 **Ans.**

For squared and ground ends, the total number of turns,

$n' = 14 + 2 = 16$ **Ans.**

4- Free length of the spring.

$$L_F = n'.d + \delta + 0.15\delta$$

$$L_F = 16 \times 6.3 + 25 + 0.15 \times 15$$
$$\therefore L_F = 129.55 \ mm$$ **Ans.**

5- Pitch of the coil.

$$p = \frac{L_F}{n' - 1} = \frac{129.55}{16 - 1} \Rightarrow p = 8.63 \ mm$$ **Ans.**

Post test

1- A compression coil spring made of an alloy steel is having the following specifications:
D = 40 mm , C = 5 , τ = 80 MPa , G = 84 kN / mm^2 , k = 23 N / mm .
Required check whether the spring under the curvature or not ?

2- Design a compression helical spring to carry a load of (500 N) with a deflection of (25 mm). The spring index may be taken as (8). Assume the following values for the spring material:
Permissible shear stress (τ) = 350 MPa
Modulus of rigidity (G) = 84 kN/mm^2
Wahl's factor $K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$

3- Design a spring for a balance to measure (0 to 1000 N). Deflection is (80 mm) and (4 mm) wire diameter. The spring is to be enclosed in a casing of (25 mm) diameter. The approximate number of turns is (30). The modulus of rigidity is (85 kN/mm^2). Also calculate the maximum shear stress induced.

Key

<u>Pre test (1)</u>

Question	Answer
1-	c
2-	d
3-	c

<u>Pre test (2)</u>

Question	Answer
1-	c
2-	b
3-	b

<u>Post test</u>

Question	Answer
1-	Spring is subjected to curvature
2-	d = 5.893 mm , D = 47.144, n = 6 , L_F = 64.096 mm , p = 12.82 mm
3-	D = 19.36 mm , L_F = 148.75 mm , p = 5.129 mm

References:-

1- A TEXTBOOK OF MACHINE DESIGN – R.S. KHURMI & J.K. GUPTA

وزارة التعليم العالي والبحث العلمي

هيئة التعليم التقني

المعهد التقني – الديوانية

حقيبة رقم (7)

وحده نمطية لدراسة (*The Belts*)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة وسائل نقل الحركة

(السيور والبكرات, أنواعها, استخداماتها, حساباتها)

إعداد

المدرس

فائق حامد جبوري

(7) The belts

7 – 1 Introduction

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

The amount of power transmitted depends upon the following factors:-
1- The velocity of the belt.
2- The tension under which the belt is placed on the pulleys.
3- The arc of contact between the belt and the smaller pulley.
4- The conditions under which the belt is used.

7 – 2 Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:
1- Speed of the driving and driven shafts,
2- Speed reduction ratio,
3- Power to be transmitted,
4- Centre distance between the shafts,
5- Positive drive requirements,
6- Shafts layout,
7- Space available, and
8- Service conditions.

7 – 3 Types of Belt Drives

The belt drives are usually classified into the following three groups:

1- Light drives

These are used to transmit small powers at belt speeds up to about (10 m/s) as in agricultural machines and small machine tools.

2- Medium drives

These are used to transmit medium powers at belt speeds over (10 m/s) but up to (22 m/s), as in machine tools.

3- Heavy drives

These are used to transmit large powers at belt speeds above (22 m/s) as in compressors and generators.

7 – 4 Types of Belts

Though there are many types of belts used these days, yet the following are important from the subject point of view:

1- Flat belt

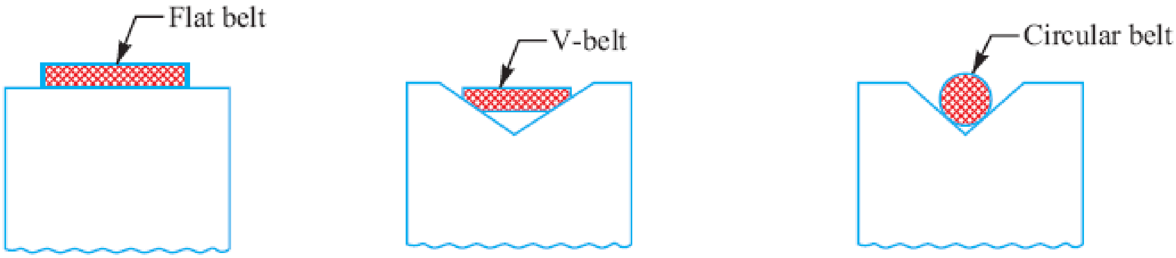
The flat as shown in Fig. (7 – 1. *a*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than (8 metres) apart.

2- V- belt

The V-belt as shown in Fig. (7 – 1. *b*), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

3- Circular belt or rope

The circular belt or rope as shown in Fig. (7 – 1 . *c*) is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 metres apart.



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

Fig. (7 – 1). Types of belts

If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

7 – 5 Types of Flat Belt Drives

The power from one pulley to another may be transmitted by any of the following types of belt drives.

1- Open belt drive

The open belt drive, as shown in Fig. (7 – 2), is used with shafts arranged parallel and rotating in the same direction. In this case, the driver A pulls the belt from one side (i.e. lower side RQ) and delivers it to the other side (i.e. upper side LM). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as **tight side** whereas the upper side belt (because of less tension) is known as **slack side,** as shown in Fig. (7 – 2).

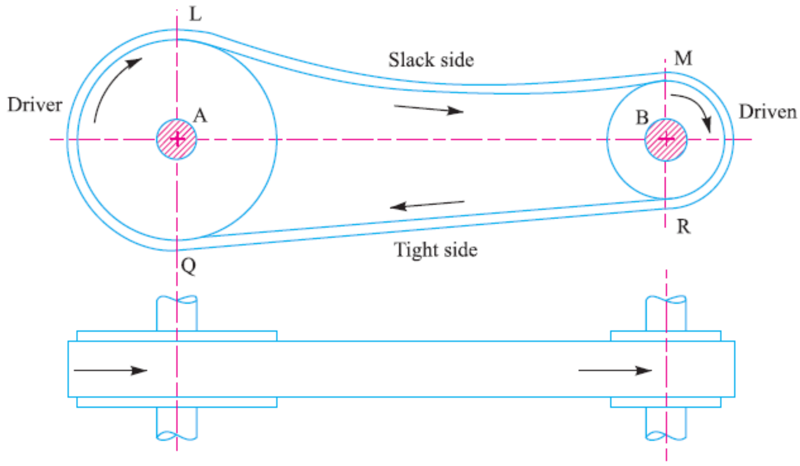


Fig. (7 – 2). Open belt drive.

2- Crossed or twist belt drive

The crossed or twist belt drive, as shown in Fig. (7 – 3), is used with shafts arranged parallel and rotating in the opposite directions. In this case, the driver pulls the belt from one side (i.e. RQ) and delivers it to the other side (i.e. LM). Thus, the tension in the belt RQ will be more than that in the belt LM. The belt RQ (because of more tension) is known as **tight side,** whereas the belt LM (because of less tension) is known as **slack side,** as shown in Fig. (7 – 3).

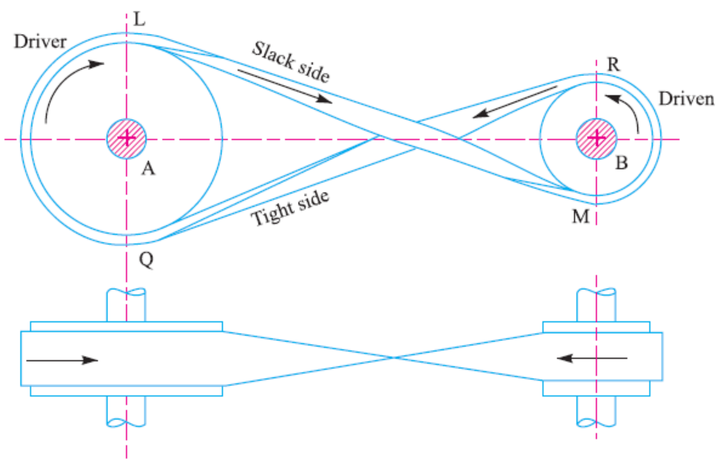
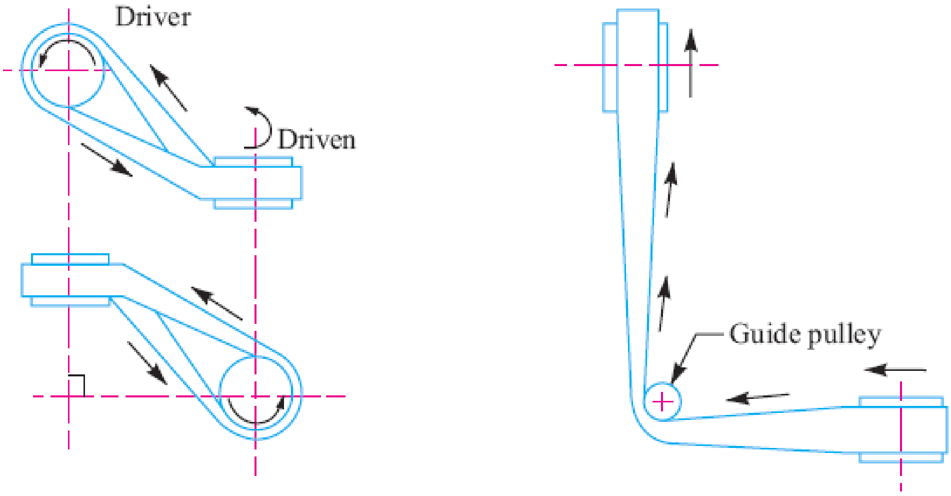


Fig. (7 – 3). Crossed or twist belt drive.

A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a maximum distance of (20 *b*), where (*b*) is the width of belt and the speed of the belt should be less than (15 m/s).

3- Quarter turn belt drive

The quarter turn belt drive (also known as **right angle belt drive**) as shown in Fig. (7 – 4 . a), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to (1.4 b), where b is width of belt. In case the pulleys cannot be arranged as shown in Fig. (7 – 4 . a) or when the reversible motion is desired, then a **quarter turn belt drive with a guide pulley,** as shown in Fig. (7 – 4 . b), may be used.

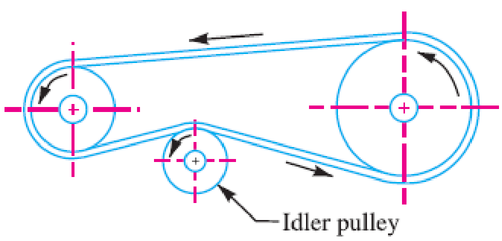


(*a*) Quarter turn belt drive. **(*b*) Quarter turn belt drive with guide pulley.**

Fig. (7 – 4)

4- Belt drive with idler pulleys.

A belt drive with an idler pulley (also known as **jockey pulley drive**) as shown in Fig. (7 – 5), is used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension cannot be obtained by other means.



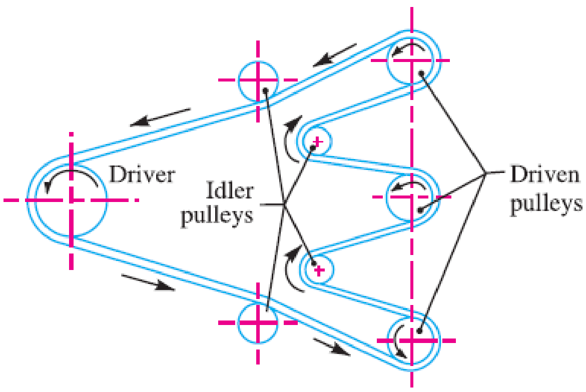


Fig. (7 – 5). Belt drive with single idler pulley . Fig. (7 – 6). Belt drive with many idler pulleys.

When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig. (7 – 6), may be employed.

5- Compound belt drive

A compound belt drive as shown in Fig. (7 – 7), is used when power is transmitted from one shaft to another through a number of pulleys.

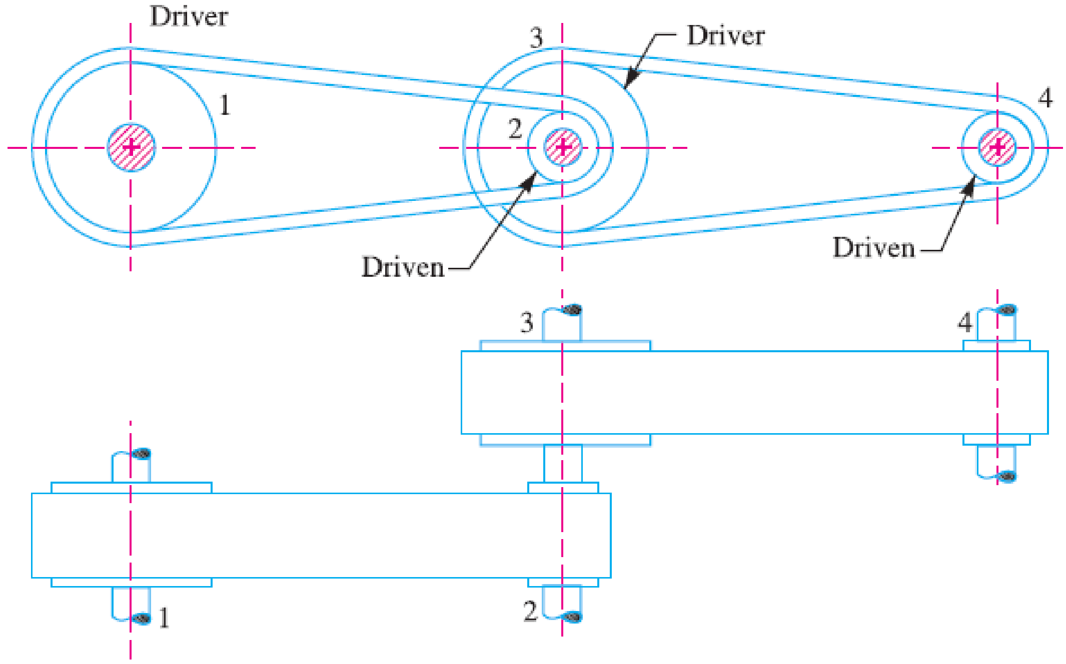


Fig. (7 – 7). Compound belt drive.

6- Stepped or cone pulley drive.

A stepped or cone pulley drive, as shown in Fig. (7 – 8), is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.

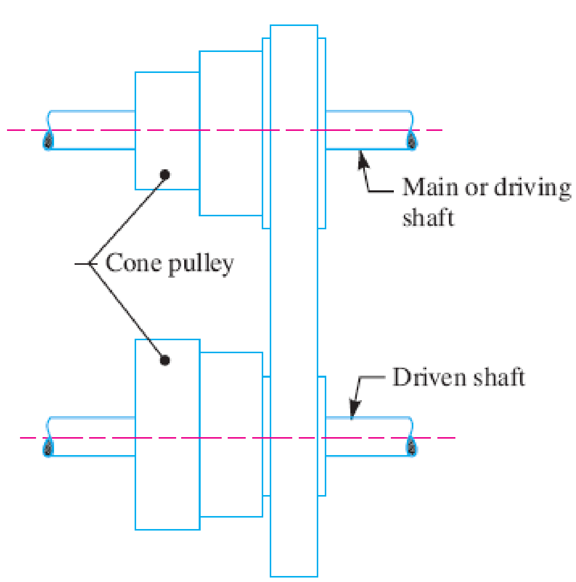


Fig. (7 – 8). Stepped or cone pulley drive.

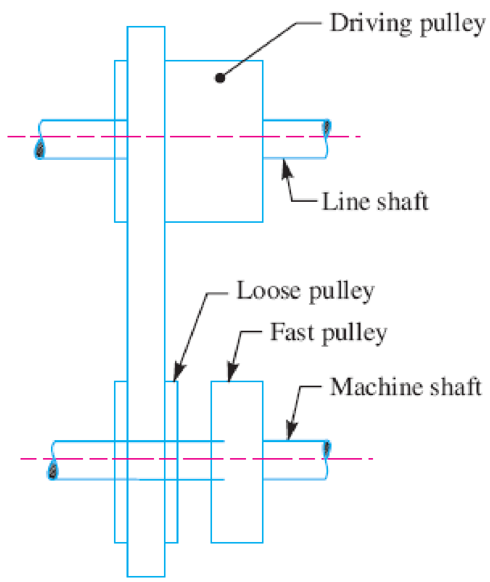


Fig. (7 – 9). Fast and loose pulley drive.

7- Fast and loose pulley drive.

A fast and loose pulley drive, as shown in Fig. (7 – 9), is used when the driven or machine shaft is to be started or stopped whenever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called fast pulley and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.

7 – 6 Velocity Ratio of a Belt Drive

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below:

Let d_1 = Diameter of the driver,

d_2 = Diameter of the follower,

N_1 = Speed of the driver in r.p.m.,

N_2 = Speed of the follower in r.p.m.,

$\therefore$ Length of the belt that passes over the driver, in one minute $= \pi\ d_1 N_1$

Similarly, length of the belt that passes over the follower, in one minute $= \pi\ d_2 N_2$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$$\because \quad \pi\ d_1 N_1 = \pi\ d_2 N_2$$

And velocity ratio, $\dfrac{N_2}{N_1} = \dfrac{d_1}{d_2}$

When thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Notes :-

1- The velocity ratio of a belt drive may also be obtained as discussed below:

We know that the peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi\ d_1 N_1}{60} \quad m/s$$

and peripheral velocity of the belt on the driven pulley,

$$v_2 = \frac{\pi\ d_2 N_2}{60} \quad m/s$$

When there is no slip, then $v_1 = v_2$

$$\therefore \quad \frac{\pi\ d_1N_1}{60} = \frac{\pi\ d_1N_1}{60} \qquad \text{or} \qquad \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

2- In case of a compound belt drive as shown in Fig. (7 – 7), the velocity ratio is given by:

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$$

or
$$\frac{Speed\ of\ last\ driven}{Speed\ of\ diameter driver} = \frac{product\ of\ diameters of\ drivers}{product\ of\ diameters of\ driverns}$$

7 – 6 Slip of the Belt

In the previous articles we have discussed the motion of belts and pulleys assuming a firm frictional grip between the belts and the pulleys. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This is called **slip of the belt** and is generally expressed as a percentage. The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance (as in the case of hour, minute and second arms in a watch).

Let s_1 % = Slip between the driver and the belt, and
s_2 % = Slip between the belt and follower,

$\therefore$ Velocity of the belt passing over the driver per second,

$$v = \frac{\pi\ d_1N_1}{60} - \frac{\pi\ d_1N_1}{60} \times \frac{1_1}{100}$$

$$v = \frac{\pi\ d_1N_1}{60}\left(1 - \frac{s_1}{100}\right) \qquad \text{---------------- (1)}$$

and velocity of the belt passing over the follower per second

$$\frac{\pi\ d_2N_2}{60} = v - v\left(\frac{s_2}{100}\right) = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation (1), we have

$$\frac{\pi\ d_2N_2}{60} = \frac{\pi\ d_1N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100} + \frac{s_1 \times s_2}{10000}\right)$$

Neglecting $\left(\frac{s_1 \times s_2}{10000}\right)$ very small

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left[1 - \left(\frac{s_1 + s_2}{100}\right)\right]$$

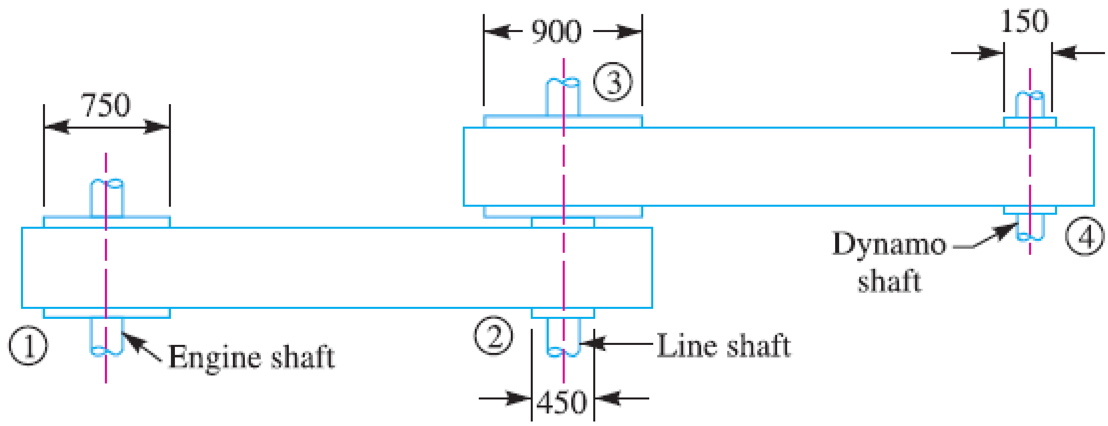
$$\therefore \frac{N_2}{N_1} = \frac{d_1}{d_2}\left[1 - \left(\frac{s}{100}\right)\right]$$ (where $s = s_1 + s_2$ i.e. total percentage of slip)

If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

Example (1):- An engine running at 150 r.p.m. drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft is 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Fine the speed of dynamo shaft, when 1. there is no slip, and 2. there is a slip of 2% at each drive.

Solution. Given : N_1 = 150 r.p.m. , d_1 = 750 mm , d_2 = 450 mm , d_3 = 900 mm d_4 = 150 mm ,$s_1 = s_2$ = 2%



All dimensions in mm.

1. When there is no slip.

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \Rightarrow N_4 = N_1 \times \frac{d_1 \times d_3}{d_2 \times d_4}$$

$$\therefore \; N_4 = 150 \times \frac{750 \times 900}{450 \times 150} = 1500 \; r.p.m.$$ **Ans.**

2. When there is a slip of 2% at each drive.

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s}{100}\right) \quad \Rightarrow N_4 = N_1 \times \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1 + s_2}{100}\right)$$

$$\therefore \; N_4 = 150 \times \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2+2}{100}\right) = 1440 \; r.p.m.$$ **Ans.**

7 – 7 Length of an Open Belt Drive

Let the belt leaves the larger pulley at *E* and *G* and the smaller pulley at *F* and *H* as shown in Fig. (7 – 10). Through O_2 draw O_2M parallel to *FE*.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,
x = Distance between the centres of two pulleys (*i.e.* O_1O_2), and
L = Total length of the belt.

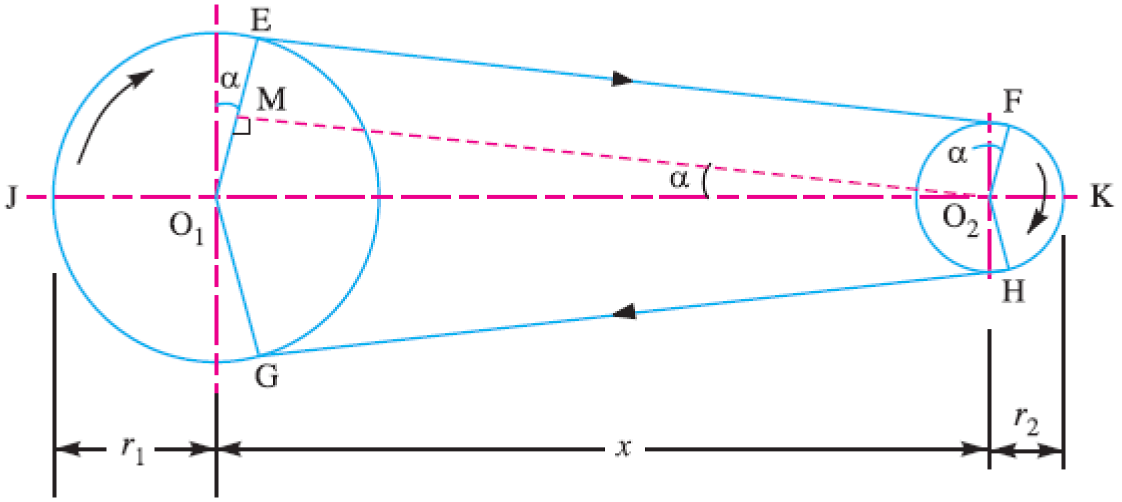


Fig. (7 – 10). Open belt drive.

$$L = \pi\ (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$ (in terms of pulley radii)

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$ (in terms of pulleys diameters)

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2}$$

$$\therefore \sin\alpha = \frac{r_1 - r_2}{x}$$

7 – 8 Length of a Cross Belt Drive

Let the belt leaves the larger pulley at *E* and *G* and the smaller pulley at *F* and *H* as shown in Fig. (7 – 11).

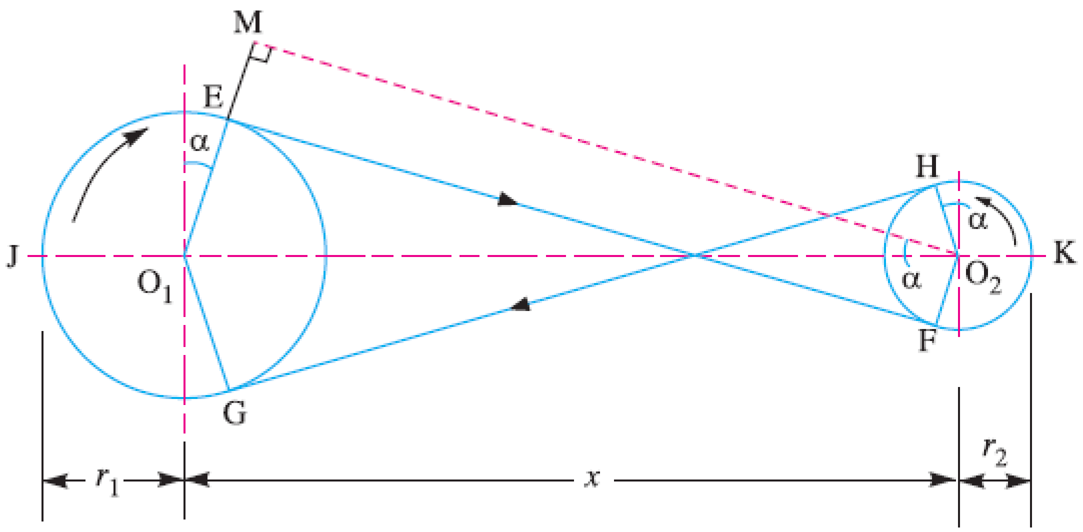


Fig. (7 – 11). Crossed belt drive.

$$L = \pi\ (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$ (in terms of pulley radii)

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}$$ (in terms of pulleys diameters)

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2}$$

$$\therefore \sin\alpha = \frac{r_1 + r_2}{x}$$

7 – 9 Power Transmitted by a Belt

Fig. (7 – 12) shows the driving pulley (or driver) *A* and the driven pulley (or follower) *B*. As already discussed, the driving pulley pulls the belt from one side and delivers it to the other side. It is thus obvious that the tension on the former side (*i.e.* tight side) will be greater than the latter side (*i.e.* slack side) as shown in Fig. (7 – 12).

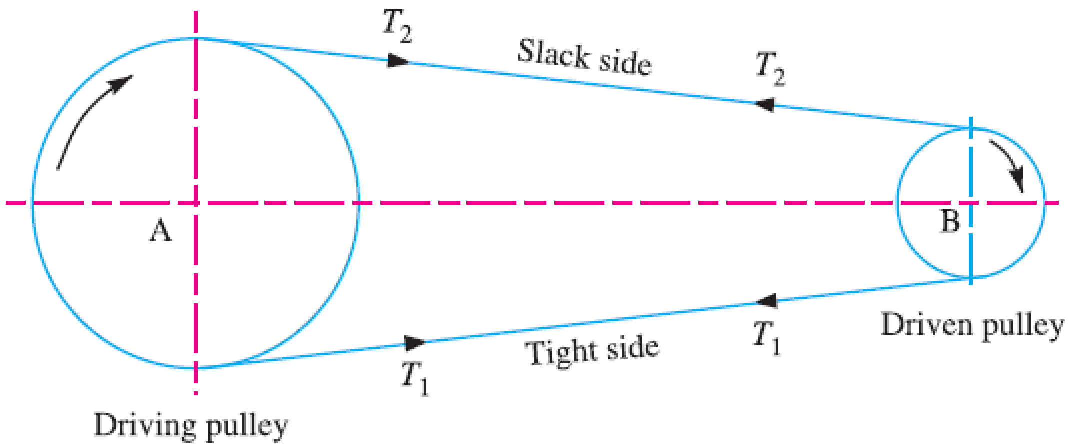


Fig. (7 – 12). Power transmitted by a belt.

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in newtons,
r_1 and r_2 = Radii of the driving and driven pulleys respectively in metres,
and v = Velocity of the belt in m/s.

The effective turning (driving) force at the circumference of the driven pulley or follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

Work done per second $= (T_1 - T_2) \times v$ N-m/s

and power transmitted (P), $P = (T_1 - T_2) \times v$ W ... ($\because$ 1 N-m/s = 1W)

A little consideration will show that torque exerted on the driving pulley is $(T_1 - T_2)\, r_1$.

Similarly, the torque exerted on the driven pulley is $(T_1 - T_2)\, r_2$.

7 – 10 Ratio of Driving Tensions for Flat Belt Drive

Let T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc *AB*, along which the belt touches the pulley at the centre).

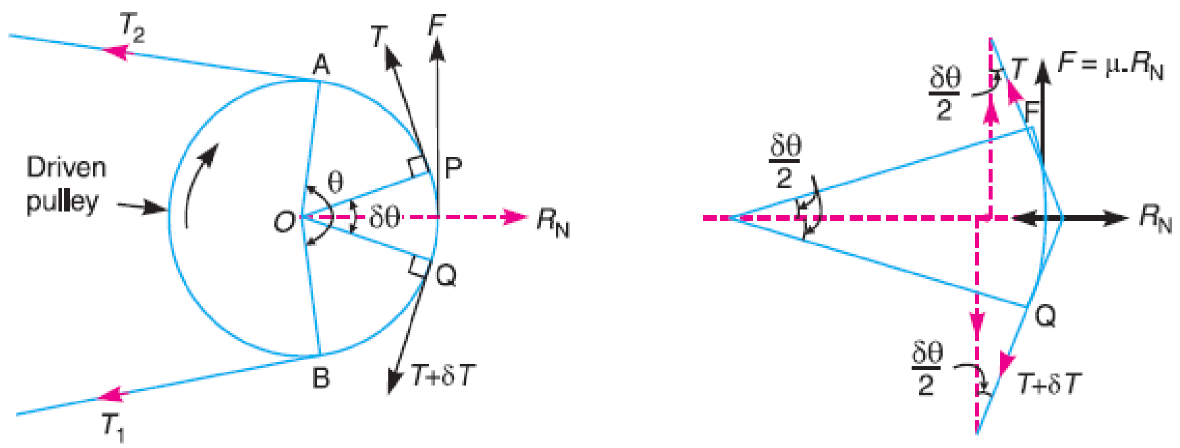


Fig. (7 – 13). Ratio of driving tensions for flat belt.

Resolving the forces in the tangential direction

$$\mu R + T\cos\frac{d\theta}{2} - (T + dT)\cos\frac{d\theta}{2} = 0$$

$$\cos\frac{d\theta}{2} = 1$$

$$\mu R + T - (T + dT) = 0$$

$$dT = \mu R \qquad \text{--------------- (1)}$$

Resolving the forces in the radial direction

$$R + T\sin\frac{d\theta}{2} - (T + dT)\sin\frac{d\theta}{2} = 0$$

$$\sin\frac{d\theta}{2} = \frac{d\theta}{2}$$

$$R + T.\frac{d\theta}{2} - (T + dT).\frac{d\theta}{2} = 0$$

$$R = Td\theta \qquad \text{-------------- (2)}$$

$$dT = \mu T d\theta$$

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\theta} \mu d\theta$$

$$\frac{T_1}{T_2} = e^{\mu.\theta} \qquad \text{------------- (3)}$$

The above expression (3) gives the relation between the tight side and slack side tensions, in terms of coefficient of friction and the angle of contact.

Notes :

1- While determining the angle of contact, it must be remembered that it is the angle of contact at the, smaller pulley, if both the pulleys are of the same material. We know that

$\therefore$ Angle of contact or lap (θ),

$$\theta = \left(180^o - 2\alpha\right)\frac{\pi}{180} \ \ rad \qquad \text{(for open belt drive)}$$

$$\theta = \left(180^o + 2\alpha\right)\frac{\pi}{180} \ \ rad \qquad \text{(for cross-belt drive)}$$

2- When the pulleys are made of different material (*i.e.* when the coefficient of friction of the pulleys or the angle of contact are different), then the design will refer to the pulley for which ($\mu.\theta$) is small.

Example (2):- Find the power transmitted by a belt running over a pulley of (600 mm) diameter at (200 r.p.m.) The coefficient of friction between the belt and the pulley is (0.25), angle of lap (160°) and maximum tension in the belt is (2500 N).

Solution.

Given: d = 600 mm = 0.6 m , N = 200 r.p.m. , $\mu = 0.25$, $T_1 = 2500$ N

$\theta = 160° = 160 \times \pi / 180 = 2.793$ rad.

We know that velocity of the belt,

$$v = \frac{\pi \ d \ N}{60} = \frac{\pi \times 0.6 \times 200}{60} \quad \Rightarrow \ v = 6.284 \ \ m/s$$

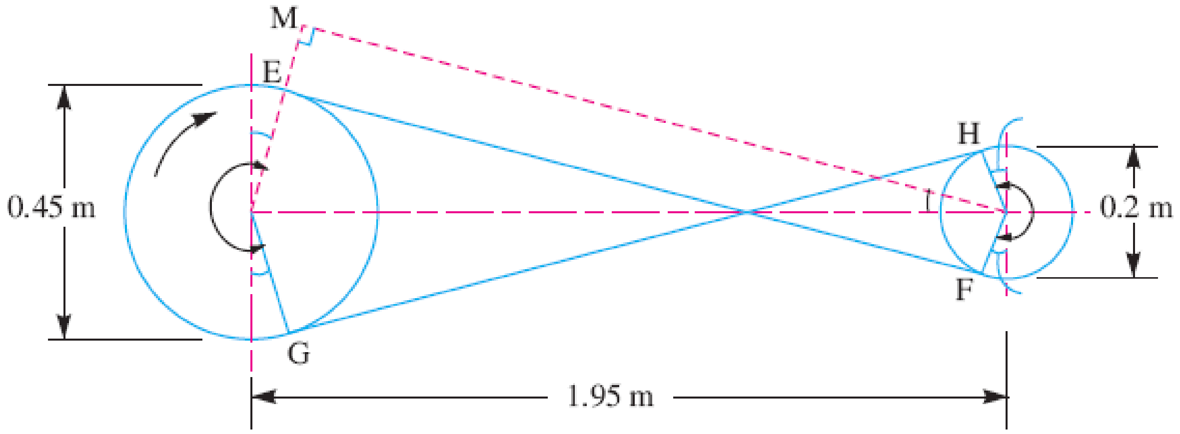
$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \Rightarrow \quad T_2 = \frac{T_1}{e^{\mu.\theta}}$$

$$\therefore \ T_2 = \frac{2500}{e^{(0.25\times 2.793)}} = 1243.78 \ \ N \approx 1244 \ \ N$$

$$P = \left(T_1 - T_2\right)\times v \quad \Rightarrow \ P = \left(2500 - 1244\right)\times 6.284$$

$\therefore P = 7892.7 \ \ W \quad = 7.89 \ \ kW$ **Ans.**

Example (3):- Two pulleys, one (450 mm) diameter and the other (200 mm) diameter, on parallel shafts (1.95 m) apart are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley. What power can be transmitted by the belt when the larger pulley rotates at (200 rev/min), if the maximum permissible tension in the belt is (1 kN), and the coefficient of friction between the belt and pulley is (0.25)?



Solution:-

Given: d_1 = 450 mm = 0.45 m , d_2 = 200 mm = 0.2 m , x = 1.95 m ,
N_1 = 200 r.p.m. , T_1 = 1 kN = 1000 N , μ = 0.25

Length of the belt

$$L = \pi \left(r_1 + r_2\right) + 2x + \frac{\left(r_1 + r_2\right)^2}{x}$$

$$L = \pi \left(0.225 + 0.1\right) + 2 \times 1.95 + \frac{\left(0.225 + 0.1\right)^2}{1.95}$$

$$\therefore \quad L = 4.974 \ m$$ **Ans.**

Angle of contact between the belt and each pulley (θ)

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.16667$$

$$\therefore \alpha = 9.6^o$$

$$\theta = \left(180^o + 2\alpha\right)\frac{\pi}{180}$$

$$\theta = \left(180^o + 29.6\right) \times \frac{\pi}{180} = 3.477 \ rad$$ **Ans.**

Power transmitted

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \Rightarrow \quad T_2 = \frac{T_1}{e^{\mu.\theta}}$$

$$T_2 = \frac{1000}{e^{(0.25 \times 3.477)}} = 419.3 \ N$$

We know that the velocity of belt,

$$v = \frac{\pi\ d_1 N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.173\ m/s$$

$\therefore$ Power transmitted

$$P = (T_1 - T_2) \times v \quad \Rightarrow \quad P = (1000 - 419.3) \times 4.173$$

$\therefore P = 2736.4\ W \quad = 2.7364\ kW$ **Ans.**

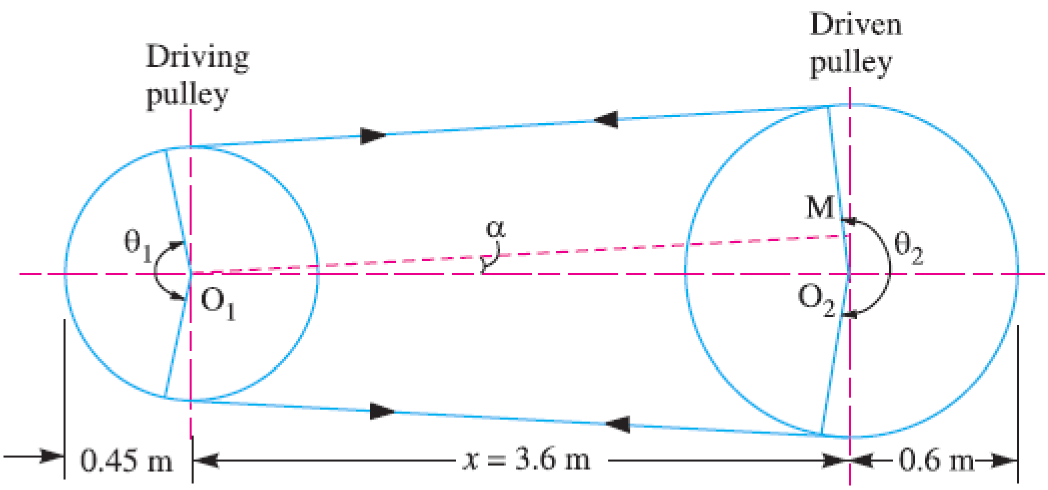
Homework:

1- A flat belt is required to transmit (30 kW) from a pulley of (1.5 m) effective diameter running at (300 r.p.m.) The angle of contact is (165^o). The coefficient of friction between the belt and pulley surface is (0.3) . Determine the maximum tension in this belt ?

2- An engine shaft running at (120 r.p.m.) is required to drive a machine shaft by means of a belt. The pulley on the engine shaft is of (2 m) diameter and that of the machine shaft is (1 m) diameter. If the belt thickness is (5 mm). Determine the speed of the machine shaft, when:-
a- there is no slip, and b- there is a slip of (3%).

3- A pulley is driven by a flat belt running at a speed of (600 m/min). The coefficient of friction between the pulley and the belt is (0.3) and the angle of lap is (160°). If the maximum tension in the belt is (700 N). Find the power transmitted by a belt.

4- A belt drive to transmit (35 kW) for a system consisting of two pulleys as shown in the Fig. below, a belt speed (20 m /s) and maximum tension is (4000 N). Find:-
a- length of belt,
b- Coefficient of friction, and
c- Speed of driven pulley when there is a slip of (2%).



وزارة التعليم العالي والبحث العلمي

هيئة التعليم التقني

المعهد التقني – الديوانية

حقيبة رقم (8)

وحده نمطية لدراسة (*The Gear And Gear Train*)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة التروس وأنواعها مع حسابات تصميم التروس العدلة وحساب القوى المؤثرة عليها باستخدام معادلة لويس ودراسة أنظمة تعشيق التروس

إعداد

المدرس

فائق حامد جبوري

CHAPTER (8)

The Gear And Gear Tran

8 – 1 Introduction

The slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by means of **gears** or **toothed wheels.** A gear drive is also provided, when the distance between the driver and the follower is very small.

8 – 2 Advantages and Disadvantages of Gear Drives

The following are the advantages and disadvantages of the gear drive as compared to other drives, i.e. belt, rope and chain drives:

Advantages

1- It transmits exact velocity ratio.
2- It may be used to transmit large power.
3- It may be used for small centre distances of shafts.
4- It has high efficiency.
5- It has reliable service.
6- It has compact layout.

Disadvantages

1- Since the manufacture of gears require special tools and equipment, therefore it is costlier than other drives.
2- The error in cutting teeth may cause vibrations and noise during operation.
3- It requires suitable lubricant and reliable method of applying it, for the proper operation of gear drives.

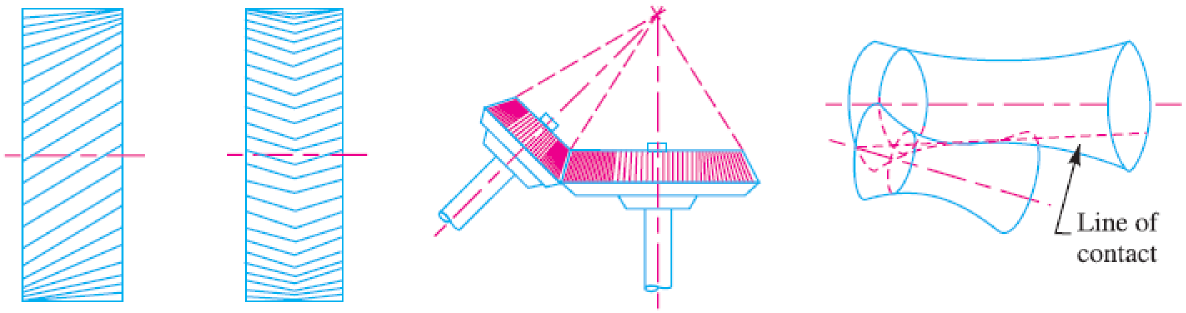
8 – 3 Classification of Gears

The gears or toothed wheels may be classified as follows:

1- According to the position of axes of the shafts

The axes of the two shafts between which the motion is to be transmitted, may be:-

a- Parallel, b- Intersecting, and c- Non-intersecting and non-parallel.



(*a*) Single helical gear. (*b*) Double helical gear. (*c*) Bevel gear. (*d*) Spiral gear.

Fig. (8 – 1)

2- According to the peripheral velocity of the gears

The gears, according to the peripheral velocity of the gears may be classified as:

1- Low velocity, 2- Medium velocity, and 3- High velocity.

The gears having velocity less than (3 m/s) are termed as low velocity gears and gears having velocity between (3 and 15 m/s) are known as medium velocity gears. If the velocity of gears is more than (15 m/s), then these are called high speed gears.

Fig. (8 – 2)

3- According to the type of gearing

The gears, according to the type of gearing may be classified as :

1- External gearing, 2- Internal gearing, and 3- Rack and pinion.

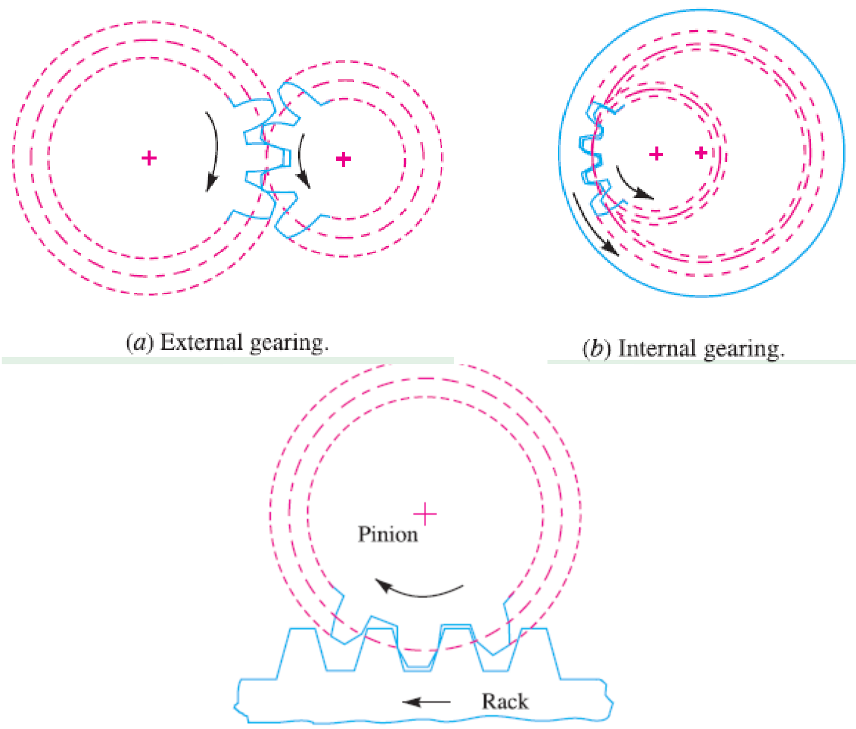


Fig. (8 – 3)

8 – 4 Technical terms

1- Circular pitch (p_c)

It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth.

$$p_c = \frac{\pi \ D}{Z}$$

Where:- D = Diameter of the pitch circle, and
Z = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note:

If (D_1) and (D_2) are the diameters of the two meshing gears having the teeth (Z_1) and (Z_2) respectively, then for them to mesh correctly,

$$p_c = \frac{\pi \ D_1}{Z_1} = \frac{\pi \ D_2}{Z_2} \quad \text{or} \quad = \frac{D_1}{Z_1} = \frac{D_2}{Z_2}$$

2- Diametral pitch (p_d)

It is the ratio of number of teeth to the pitch circle diameter in millimetres.

$$p_d = \frac{Z}{D} = \frac{\pi}{p_c}$$

Where:- Z = Number of teeth, and D = Pitch circle diameter.

3-Module (*m*)

It is the ratio of the pitch circle diameter in millimetres to the number of teeth.

$$m = \frac{D}{Z}$$

$$p_c \times p_d = \frac{\pi\ D}{Z} \times \frac{Z}{D}$$

$$\therefore\ \ p_c \times p_d = \pi$$

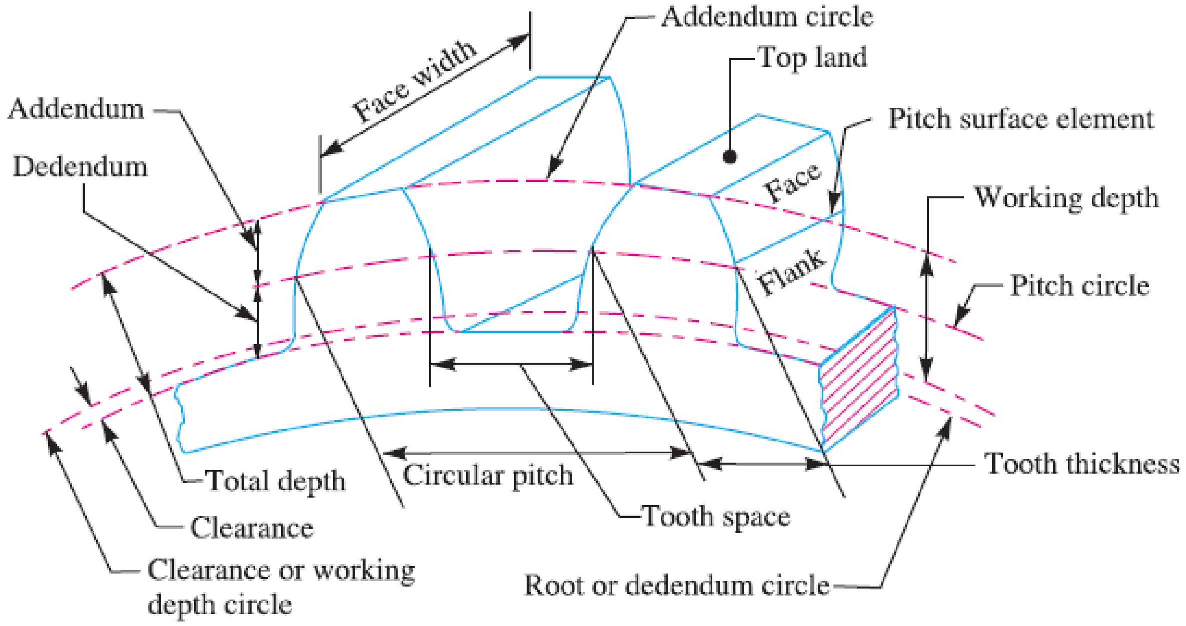


Fig. (8 – 4). Terms used in gears.

Fig. (8 – 5). Spur gears

Example (1):- A gear seat consist of (16) teeth pinion driving (40) teeth gear the diameter pitch is (2 number per cm) compute ? a- pitch circular. b- pitch circle diameter. **Solution:-** Give: $Z_1 = 16$ tooth , $Z_2 = 40$ tooth , $p_d = 2$ cm .

a- $$p_c \times p_d = \pi \quad \Rightarrow \quad p_c = \frac{\pi}{p_d} = \frac{\pi}{2} = 1.57 \ cm$$

b- $$p_d = \frac{Z}{D} \quad \Rightarrow \quad D = \frac{Z}{p_d}$$

$$D_1 = \frac{Z_1}{p_d} = \frac{16}{2} = 8 \ cm$$

$$D_2 = \frac{Z_2}{p_d} = \frac{40}{2} = 20 \ cm$$

Example (2):- Two wheels of (18) and (45) teeth respectively are geared together. Determine the pitch circle diameter of the wheels if the circular pitch is (5 cm) ?

Solution:- Give: $Z_1 = 18$ tooth , $Z_2 = 45$ tooth , $p_c = 5$ cm .

$$p_c = \frac{\pi D}{Z} \quad \Rightarrow \quad D = \frac{p_c \times Z}{\pi}$$

$$D_1 = \frac{p_c \times Z_1}{\pi} = \frac{5 \times 18}{\pi} = 28.64 \ cm$$

$$D_2 = \frac{p_c \times Z_2}{\pi} = \frac{5 \times 45}{\pi} = 71.619 \ cm$$

8 – 5 Beam Strength of Gear Teeth – Lewis Equation

The beam strength of gear teeth is determined from an equation (known as Lewis equation) and the load carrying ability of the toothed gears as determined by this equation gives satisfactory results. In the investigation, Lewis assumed that as the load is being transmitted from one gear to another, it is all given and taken by one tooth, because it is not always safe to assume that the load is distributed among several teeth.

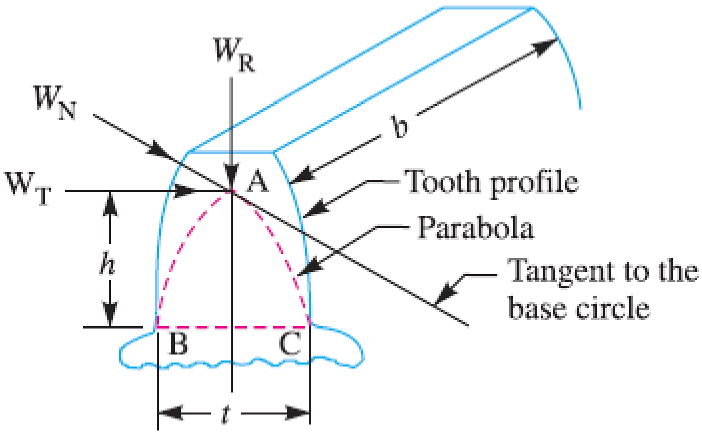


Fig. (8 – 6). Tooth of a gear.

W_R = radial component acting perpendicular and parallel to the centre line of the tooth
W_N = normal load
W_T = tangential component

The maximum value of the bending stress (or the permissible working stress σ_w)

$$\sigma_w = \frac{M \times y}{I} \qquad \text{------------- (1)}$$

Where:- M = Maximum bending moment at the critical section $BC = W_T \times h$,
W_T = Tangential load acting at the tooth,
h = Length of the tooth,
y = Half the thickness of the tooth (t) at critical section $BC = t / 2$,
I = Moment of inertia about the centre line of the tooth $= b.t^3 / 12$,
b = Width of gear face.

Substituting the values for M , y and I in equation (1), we get:-

$$\sigma_w = \frac{(W_T \times h) \times t/2}{b.t^3/12} = \frac{(W_T \times h) \times 6}{b.t^2}$$

or $$W_T = \sigma_w \times b \times \frac{t^2}{6h}$$

Let $t = x \times p_c$, and $h = k \times p_c$

Where:- x and k are constants.

$$W_T = \sigma_w \times b \times \frac{x^2 . {p_c}^2}{6k . p_c} \Rightarrow W_T = \sigma_w \times b \times p_c \frac{x^2}{6k}$$

Substituting $x^2 / 6k = y$, another constant, we have

$$W_T = \sigma_w \times b \times p_c \times y \qquad \because p_c = \pi . m$$

$$\therefore W_T = \sigma_w \times b \times \pi \times m \times y$$

or $$W_T = \sigma_o \times b \times \pi \times m \times y \times C_v$$

where:- σ_o = Allowable static stress, and

C_v = Velocity factor.

The quantity y is known as Lewis form factor or tooth form factor and (W_T) (which is the tangential load acting at the tooth) is called the beam strength of the tooth.

y = form factor or tooth form factor.

Since $$y = \frac{x^2}{6k} = \frac{t^2}{(p_c)^2} \times \frac{p_c}{6h} = \frac{t^2}{6h . p_c}$$

The value of (y) in terms of the number of teeth may be expressed as follows :

$$y = 0.124 - \frac{0.684}{Z}$$ for (14.5^o) composite and full depth involutes system.

$$y = 0.124 - \frac{0.912}{Z}$$ for (20^o) full depth involutes system.

$$y = 0.124 - \frac{0.841}{Z}$$ for (20^o) stub system.

The values of the velocity factor (C_v) are given as follows :

$$C_v = \frac{3}{3 + v}$$ for ordinary cut gears operating at velocities upto (12.5 m / s)

$C_v = \dfrac{4.5}{4.5+v}$ for carefully cut gears operating at velocities upto (12.5 m / s)

$C_v = \dfrac{6}{6+v}$ for very accurately cut and ground metallic gears operating at velocities

upto (20 m / s)

$C_v = \dfrac{0.75}{0.75+\sqrt{v}}$ for precision gears cut with high accuracy and operating velocities

upto (20 m / s)

$C_v = \left(\dfrac{0.75}{0.75+v}\right) + 0.25$ for non – metallic gears .

In the above expressions, (*v*) is the pitch line velocity in metres per second.

Example (3):- A bronze spur pinion rotating at (600 r.p.m.) drives a cast iron spur gear. The allowable static stress for the bronze pinion (84 MPa). The bronze has (16) teeth of module (8 mm). The width of gears is (90 mm). Find the power that can be transmitted?

Take $y_P = 0.154 \times \dfrac{0.912}{Z_P}$ $\quad C_V = 0.427$

Solution:- Give: N = 600 r.p.m. , σ_o = 84 MPa , Z = 16 , m = 8 , b = 90 mm .

$$D = m \times Z = 8 \times 16 = 128 \ mm$$

$$V = \frac{\pi \times D \times N}{60} = \frac{\pi \times 0.128 \times 600}{60} = 4.02 \ m/s$$

$$y_P = 0.154 \times \frac{0.912}{Z_P} \quad \Rightarrow y_P = 0.154 \times \frac{0.912}{16} = 0.097$$

$$W_T = \sigma_o \times b \times \pi \times m \times y_P \times C_V$$

$$W_T = 84 \times 90 \times \pi \times 8 \times 0.097 \times 0.427 = 7869.755 \ N$$

$$P = W_T \times V = 7869.755 \times 4.02 = 31636.4 \ W \quad = 31.6364 \ kW$$

8 – 6 Gear Train

1- Introduction

Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called **gear train** or **train of toothed wheels.** The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

2- Types of Gear Trains

Following are the different types of gear trains, depending upon the arrangement of wheels:

1- Simple gear train

When there is only one gear on each shaft, as shown in Fig. (8 – 7), it is known as simple gear train. The gears are represented by their pitch circles.

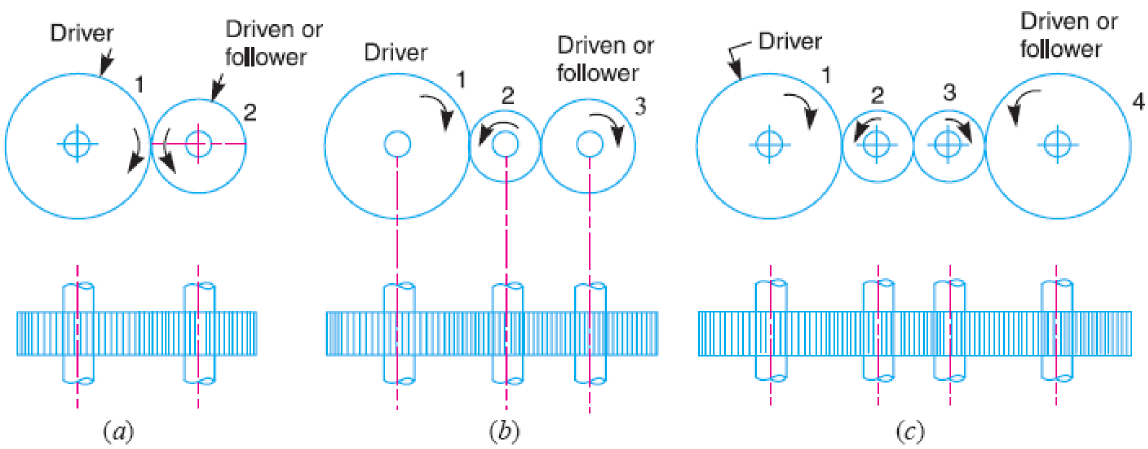


Fig. (8 – 7). Simple gear train.

2- Compound gear train

When there are more than one gear on a shaft, as shown in Fig. (8 – 8), it is called a compound train of gear.

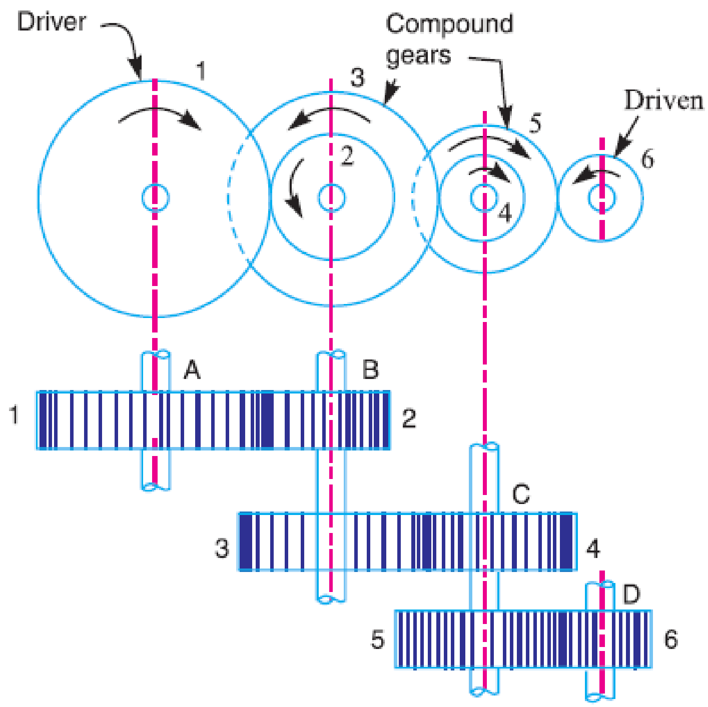


Fig. (8 – 8). Compound gear train.

3- Reverted gear train

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train as shown in Fig. (8 – 9).

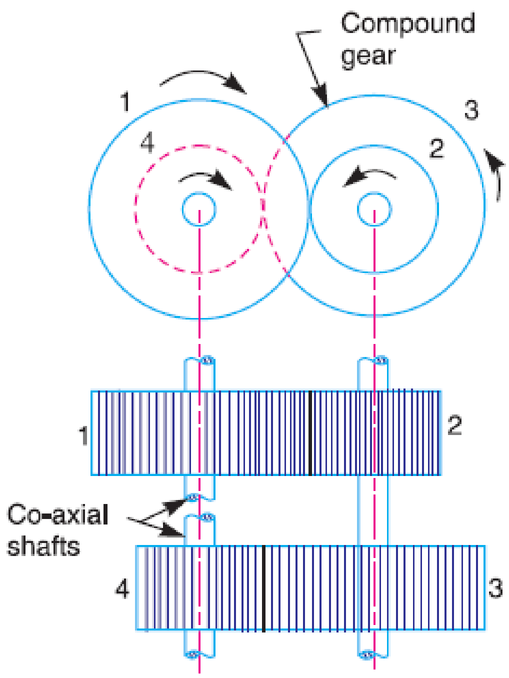


Fig. (8 – 9). Reverted gear train.

4- Epicyclic gear train

We have already discussed that in an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. (8 – 10)

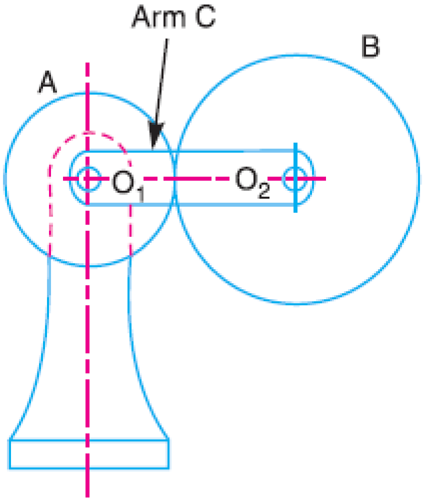


Fig. (8 – 10). Epicyclic gear train.

8 – 7 velocity ratio (V.R)

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ration } (V.R) = \frac{N_1}{N_2} = \frac{Z_1}{Z_2}$$

$$V.R = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$ for two gears train

Where:- N_1 = Speed of driver in r.p.m.,
N_2 = Speed of intermediate gear in r.p.m.

and more than two gear train ($V.R$) =

$$\frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}} = \frac{\text{product of the number of teeth on the drivens}}{\text{product of the number of teeth on the drivers}}$$

In Reverted gear train :-

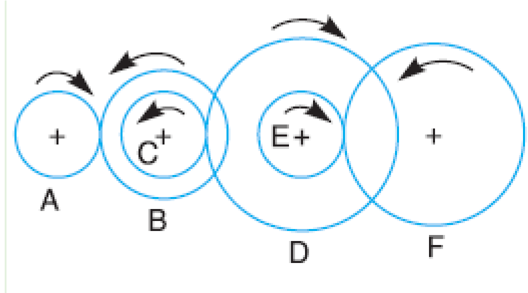
1- $r_1 + r_2 = r_3 + r_4$

2- $Z_1 + Z_2 = Z_3 + Z_4$ and ,

3- $$V.R = \frac{product\ of\ number of\ teeth\ on\ drivens}{product\ of\ number of\ teeth\ on\ drivers}$$

or $$\frac{N_1}{N_4} = \frac{Z_2 \times Z_4}{Z_1 \times Z_1}$$

Example (4):- The gearing of a machine tool is shown in Fig. below. The motor shaft is connected to gear A and rotates at (975 r.p.m.) .The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear (F)? The number of teeth on each gear are as given below:-



Gear	*A*	*B*	*C*	*D*	*E*	*F*
No. of teeth	*20*	*50*	*25*	*75*	*26*	*65*

Solution:-

$$\frac{N_A}{N_F} = \frac{Z_B \times Z_D \times Z_F}{Z_A \times Z_C \times Z_E}$$

$$N_F = N_A \times \frac{Z_B \times Z_D \times Z_F}{Z_A \times Z_C \times Z_E} = \frac{50 \times 75 \times 65}{20 \times 25 \times 26}$$

$\therefore N_F = 52\ r.p.m.$ **Ans.**

Example (5):- Two spur gear have a velocity ratio of (1/3). The drive gear has (72) tooth of (80 mm) module and rotates at (300 rpm). Calculate the number of teeth and the speed of the driver ?

Solution:- Give : $V.R = \frac{1}{3}$, $Z_1 = 72$, $N_2 = 300$ r.p.m.

$$V.R = \frac{N_2}{N_1} = \frac{Z_1}{Z_2} = \frac{1}{3}$$

$$\frac{Z_1}{72} = \frac{1}{3} \quad \Rightarrow Z_1 = 24 \ tooth$$

$$\frac{300}{N_1} = \frac{1}{3} \quad \Rightarrow N_1 = 900 \ \ r.p.m.$$

8 – 8 Design of Spur Gears

Sometimes, the spur gears (i.e. driver and driven) are to be designed for the given velocity ratio and distance between the centres of their shafts.

Let x = Distance between the centres of two shafts,
N_1 = Speed of the driver,
Z_1 = Number of teeth on the driver,
D_1 = Pitch circle diameter of the driver,
N_2 , T_2 and d_2 = Corresponding values for the driven or follower, and
p_c = Circular pitch.

We know that the distance between the centres of two shafts,

$$x = \frac{D_1 + D_2}{2} \qquad \text{-----------------} \ (1)$$

and speed ratio or velocity ratio,

$$\frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{Z_2}{Z_1} \qquad \text{------------------} \ (2)$$

From the above equations, we can conveniently find out the values of (D_1) and (D_2) or (Z_1) and (Z_1) and the circular pitch (p_c). The values of (Z_1) and (Z_2), as obtained above, may or may not be whole numbers. But in a gear since the number of its teeth is always a whole number, therefore a slight alterations must be made in the values of (*x*, D_1 and D_2), so that the number of teeth in the two gears may be a complete number.

Example (6):- Two parallel shafts, about (600 mm) apart are to be connected by spur gears. One shaft is to run at (360 r.p.m.) and the other at (120 r.p.m.). Design the gears, if the circular pitch is to be (25 mm).
Solution:- $x = 600$ mm , $N_1 = 360$ r.p.m. , $N_2 = 120$ r.p.m. , $p_c = 25$ mm

We know that speed ratio,

$$\frac{N_1}{N_2} = \frac{D_2}{D_1} \Rightarrow \frac{D_2}{D_1} = \frac{360}{120} = 3$$

$$\therefore\ D_2 = 3 \times D_1 \qquad \text{---------------} \ (\ 1\ )$$

and centre distance between the shafts (*x*) ,

$$x = \frac{D_1 + D_2}{2} \Rightarrow x = \frac{1}{2}\left(D_1 + D_2\right) \Rightarrow 600 = \frac{1}{2}\left(D_1 + D_2\right)$$

$$\therefore\ D_1 + D_2 = 1200 \qquad \text{---------------} \ (\ 2\ )$$

From equations (1) and (2), we find that

$$D_2 = 900 \ \ mm$$

$\therefore$ Number of teeth on the first gear,

$$p_c = \frac{\pi \times D}{Z} \Rightarrow Z_1 = \frac{\pi \times D_1}{p_c}$$

$$Z_1 = \frac{\pi \times 300}{25} = 37.7$$

and number of teeth on the second gear,

$$Z_2 = \frac{\pi \times D_2}{p_c} = \frac{\pi \times 900}{25} = 113.3$$

Since the number of teeth on both the gears are to be in complete numbers, therefore let us make the number of teeth on the first gear as (38) .

$$V.R = \frac{N_1}{N_2} = \frac{900}{300} = 3 \quad \Rightarrow \frac{Z_2}{Z_1} = 3$$

the number of teeth on the second gear should be ,

$$Z_2 = 3 \times 38 = 114$$

Now the exact pitch circle diameter of the first gear,

$$D_1' = \frac{Z_1 \times p_c}{\pi} = \frac{38 \times 25}{\pi} = 302.394 \ mm$$ **Ans.**

and the exact pitch circle diameter of the second gear,

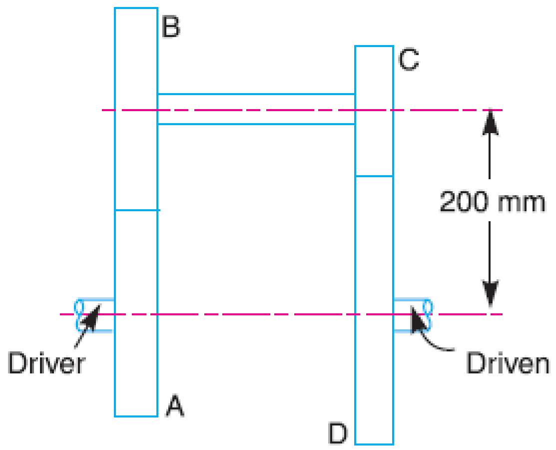
$$D_2' = \frac{Z_2 \times p_c}{\pi} = \frac{114 \times 25}{\pi} = 907.183 \ mm$$ **Ans.**

$\therefore$ Exact distance between the two shafts,

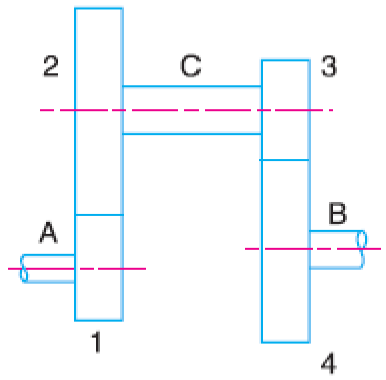
$$x' = \frac{D_1' + D_2'}{2} = \frac{302.394 + 907.183}{2} = 603.266 \ mm$$ **Ans.**

Homework:

1- The speed ratio of the reverted gear train, as shown in Fig. below, velocity ratio is (12). The module pitch of gears (A) and (B)is (3.125 mm) and of gears (C) and (D) is (2.5 mm). Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth.



2- In a reverted gear train, as shown in Fig. below, two shafts (A) and (B) are in the same straight line and are geared together through an intermediate parallel shaft (C). The gears connecting the shafts (A) and (C) have a module of (2 mm) and those connecting the shafts (C) and (B) have a module of (4.5 mm). The speed of shaft (A) is to be about but greater than (12 times) the speed of shaft (B), and the ratio at each reduction is same. Find suitable number of teeth for gears. The number of teeth of each gear is to be a minimum but not less than (16). Also find the exact velocity ratio and the distance of shaft (C) from (A) and (B).



وزارة التعليم العالي والبحث العلمي

هيئة التعليم التقني

المعهد التقني – الديوانية

حقيبة رقم (9)

وحده نمطية لدراسة (*Shafts*)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة الأعمدة وانواعها وحسابتها

إعداد

المدرس

فائق حامد جبوري

CHAPTER (9)

Shafts

9 – 1 Introduction

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it.

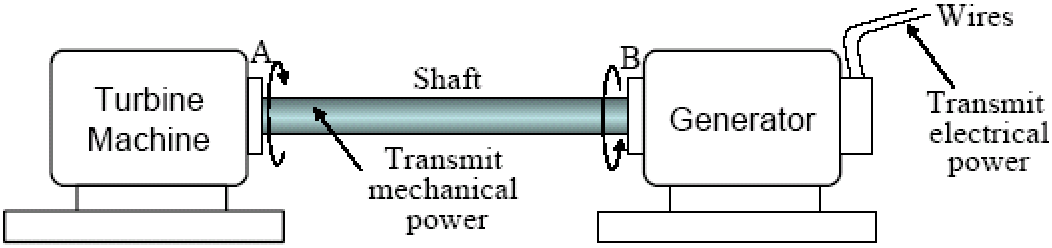


Fig. (9 – 1)

Notes:

1- The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.
2- An axle, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.
3- A spindle is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (e.g. lathe spindles).

9 – 2 Material Used for Shafts

The material used for shafts should have the following properties :

1- It should have high strength.
2- It should have good machinability.
3- It should have low notch sensitivity factor.
4- It should have good heat treatment properties.
5- It should have high wear resistant properties.

9 – 3 Manufacturing of Shafts

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses. The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

9 – 4 Types of Shafts

The following two types of shafts are important from the subject point of view:

1- Transmission shafts

These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2- Machine shafts

These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

9 – 5 Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are:-
(25 mm) to (60 mm) with (5 mm) steps,
(60 mm) to (110 mm) with (10 mm) steps,
(110 mm) to (140 mm) with (15 mm) steps,
and (140 mm) to (500 mm) with (20 mm) steps.
The standard length of the shafts are (5 m), (6 m) and (7 m).

9 – 6 Stresses in Shafts

The following stresses are induced in the shafts :

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to combined torsional and bending loads.

9 – 7 Design of Shafts

The shafts may be designed on the basis of

1- Strength, and 2- Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:-

a- Shafts subjected to twisting moment or torque only,
b- Shafts subjected to bending moment only,
c- Shafts subjected to combined twisting and bending moments, and
d- Shafts subjected to axial loads in addition to combined torsional and bending loads.

We shall now discuss the above cases, in detail, in the following pages.

a- Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation.

We know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad \text{---------------- (1)}$$

Where:- T = Twisting moment (or torque) acting upon the shaft,
J = Polar moment of inertia of the shaft about the axis of rotation,
τ = Torsional shear stress, and
r = Distance from neutral axis to the outer most fibre
= d / 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (1) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \qquad \text{or} \qquad T = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{---------------- (2)}$$

or $$d = 1.73 \times \sqrt[3]{\frac{T}{\tau}}$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

Where:- do and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation (1), we have

$$\frac{T}{\frac{\pi}{32}\left[(d_0)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \qquad \text{or} \qquad T = \frac{\pi}{16} \times \tau \times \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad \text{----------- (3)}$$

Let k = Ratio of inside diameter and outside diameter of the shaft

$$k = \frac{d_i}{d_o}$$

Now the equation (3) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times (d)^3\left[1 - k^4\right] \quad \text{----------} \ (4)$$

or $$d = 1.73 \times \sqrt[3]{\frac{T}{\tau \times \left(1 - k^4\right)}}$$

Notes:-

1- The twisting moment (T) may be obtained by using the following relation :
We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi.N.T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi.N}$$

Where:- T = Twisting moment in N-m, and
N = Speed of the shaft in r.p.m.

2- In case of belt drives, the twisting moment (T) is given by

$$T = \left(T_1 - T_2\right) \times R$$

Where:-
T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and
R = Radius of the pulley.

Example (1):- A line shaft rotating at (200 r.p.m). is to transmit (20 kW). The shaft may be assumed to be made of mild steel with an allowable shear stress of (42 MPa). Determine the diameter of the shaft, neglecting the bending moment on the shaft.

Solution:- Given: N = 200 r.p.m. , P = 20 kW = 20 × 103 W, τ = 42 MPa = 42 N/mm^2 .

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi.N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \ N.\ m$$

$$\therefore T = 955 \times 10^3 \, N.\ mm$$

We also know that torque transmitted by the shaft (T),

$$T = \frac{\pi}{16} \times \tau \times d^3 \Rightarrow 955 \times 10^3 = \frac{\pi}{16} \times 42 \times d^3$$

$$\therefore d = 48.74 \ mm \quad \text{say} \quad 50 \ mm$$ **Ans.**

b- Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I}=\frac{\sigma_b}{y} \qquad \text{-------------- (1)}$$

Where:- M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I=\frac{\pi}{64}\times d^4 \qquad \text{and} \qquad y=\frac{d}{2}$$

Substituting these values in equation (1), we have

$$\frac{M}{\frac{\pi}{64}\times d^4}=\frac{\sigma_b}{\frac{d}{2}} \qquad \text{or} \qquad M=\frac{\pi}{32}\times\sigma_b\times d^3$$

or $$d=2.17\times\sqrt[3]{\frac{M}{\sigma}}$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I=\frac{\pi}{64}\times\left[(d_o)^4-(d_1)^4\right]$$

$$\therefore I=\frac{\pi}{64}\times(d_o)^3\left(1-k^4\right)$$

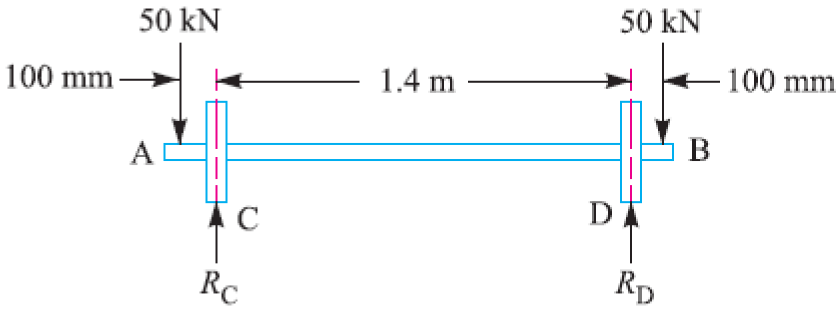
and $y = d_o / 2$

Again substituting these values in equation (1), we have

$$\frac{M}{\frac{\pi}{64}\times(d_o)^4\left(1-k^4\right)}=\frac{\sigma_b}{\frac{d_o}{2}} \qquad \text{or} \qquad M=\frac{\pi}{32}\times\sigma_b\times(d_o)^3\left(1-k^4\right)$$

or $$d=2.17\times\sqrt[3]{\frac{M}{\sigma\left(1-k^4\right)}}$$

Example (2):- A pair of wheels of a railway wagon carries a load of (50 kN) on each axle box, acting at a distance of (100 mm) outside the wheel base. The gauge of the rails is (1.4 m). Find the diameter of the axle between the wheels, if the stress is not to exceed (100 MPa).



Solution. Given: W = 50 kN = 50 × 103 N , L = 100 mm , x = 1.4 m , σ_b = 100 MPa = 100 N/mm$_2$

A little consideration will show that the maximum bending moment acts on the wheels at (C) and (D). Therefore maximum bending moment,

$$M = W \times L = 50 \times 10^3 \times 100 = 5 \times 10^6 \ N.\ mm$$

We know that the maximum bending moment (M),

$$M = \frac{\pi}{32} \times \sigma_b \times (d_o)^3 (1 - k^4) \qquad \Rightarrow \ 5 \times 10^6 = \frac{\pi}{32} \times 100 \times d^3$$

$\therefore \ d = 79.85 \ mm$ say $80 \ mm$ **Ans.**

c- Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously.

$$T_e = \frac{\pi}{16} \times \tau \times d^3$$ for solid shaft

or $$d = 1.73 \times \sqrt[3]{\frac{T_e}{\tau}}$$

Where:- T_e = equivalent twisting moment $\qquad T_e = \sqrt{M^2 + T^2}$

$$T_e = \frac{\pi}{16} \times \tau \times (d_o)^3 (1 - k^4)$$ for hollow shaft or $$d_o = 1.73 \times \sqrt[3]{\frac{T_e}{\tau (1 - k^4)}}$$

Example (3):- A solid circular shaft is subjected to a bending moment of (3000 N-m) and a torque of (10 000 N-m). The shaft is made of 45 C 8 steel having ultimate tensile stress of (700 MPa) and a ultimate shear stress of (500 MPa). Assuming a factor of safety as (6), determine the diameter of the shaft.

Solution:-

Given: M = 3000 N-m = 3 × 106 N-mm , T = 10 000 N-m = 10 × 106 N-mm , σ_{tu} = 700 MPa = 700 N/mm^2 , τ_u = 500 MPa = 500 N/mm^2

We know that the allowable tensile stress,

$$\sigma_b = \frac{\sigma_u}{F.S} = \frac{700}{6} = 116.7 \;\; N/mm^2$$

and allowable shear stress,

$$\sigma_b = \frac{\tau_u}{F.S} = \frac{500}{6} = 83.3 \;\; N/mm^2$$

$$T_e = \sqrt{M^2 + T^2} \quad \Rightarrow \quad T_e = \sqrt{\left(3\times10^6\right)^2 + \left(10\times10^6\right)^2} = 10.44\times10^6 \;\; N.\; mm$$

We also know that equivalent twisting moment (Te),

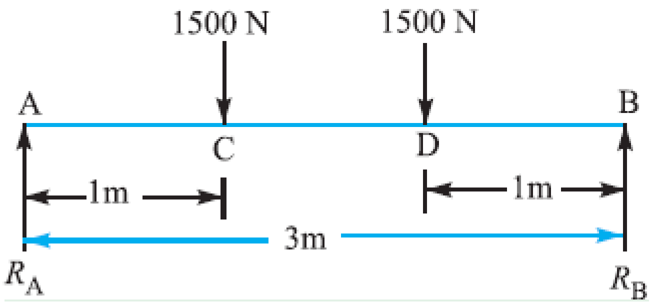
$$T_e = \frac{\pi}{16}\times\tau\times d^3 \quad \Rightarrow \quad 10.44\times10^6 = \frac{\pi}{16}\times83.333\times d^3$$

$$\therefore d = 86 \;\; mm$$

Example (4):- A shaft made of mild steel is required to transmit (100 kW) at (300 r.p.m.).The supported length of the shaft is (3 metres). It carries two pulleys each weighing (1500 N) supported at a distance of (1 metre) from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.

Solution:-

Given: $P = 100$ kW $= 100 \times 10^3$ W , $N = 300$ r.p.m. , $L = 3$ m , $W = 1500$ N



We know that the torque transmitted by the shaft,

$$T = \frac{60P}{2\pi N} \Rightarrow T = \frac{60 \times 100 \times 10^3}{2 \times \pi \times 300} = 3183 N.m$$

$$R_A = R_B = 1500 \ N$$

$$M = (1500 \times 1.5 - 1500 \times 0.5)$$

$$M = 1500 N.m$$

$$T_e = \sqrt{M^2 + T^2}$$

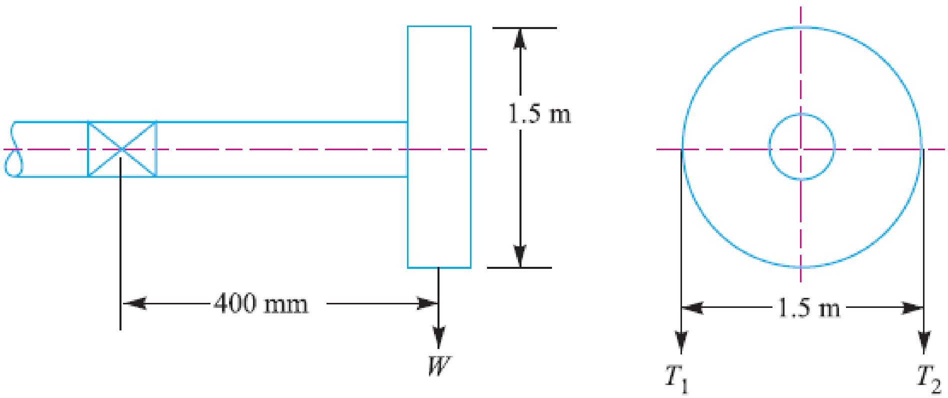
$$T_e = \sqrt{(1500)^2 + (3183)^2}$$

$$T_e = 3519 \times 10^3 \ N.mm$$

$$d = 1.72 \sqrt[3]{\frac{T_e}{\tau}} \Rightarrow d = 1.72 \sqrt[3]{\frac{3519 \times 10^3}{60}} \Rightarrow d = 66.8 mm \quad \text{say} \quad d = 70 mm$$

Example (5):- A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is (1.5 metre) in diameter and has belt tensions (5.4 kN) and (1.8 kN) on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being (400 mm), assuming maximum allowable shear stress of (42 MPa). Find:-

1- the diameter of the shaft. 2- The dimensions of key.



Solution:-

Given : D = 1.5 m or R = 0.75 m , T_1 = 5.4 kN = 5400 N, T_2 = 1.8 kN = 1800 N , L = 400 mm , τ = 42 MPa = 42 N/mm^2

1- the diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = (F_1 - F_2) \times R \quad \Rightarrow T = (5400 - 1800) \times 750$$

$$T = 2700 \times 10^3 \ N.mm$$

Neglecting the weight of shaft, total vertical load acting on the pulley, (W)

$$W = F_1 + F_2 = 5400 + 1800 = 7200 \ N$$

$\therefore$ Bending moment (M)

$$M = W \times L \quad = 7200 \times 400 = 2880 \times 10^3 \ N.mm$$

$$T_e = \sqrt{M^2 + T^2} \quad T_e = \sqrt{(2880 \times 10^3)^2 + (2700 \times 10^3)^2}$$

$$T_e = 3947.7 \times 10^3 \ N.mm$$

$$T = \frac{\pi}{16} \times \tau \times d^3 \quad \Rightarrow 3947.7 \times 10^3 = \frac{\pi}{16} \times 42 \times d^3$$

$$\therefore d^3 = \frac{3947.7 \times 10^3 \times 16}{\pi \times 42} \quad \Rightarrow d = \sqrt[3]{47870268} \quad \Rightarrow d = 78.22 \ mm$$

Say $d = 80 \ mm$ **Ans.**

2- The dimensions of key.

$$w = \frac{d}{4} = \frac{80}{4} = 20 \ mm \qquad t = \frac{d}{6} = \frac{80}{6} = 13.33 \ mm$$

$$T = l \times w \times \tau \times \frac{d}{2} \quad \Rightarrow \quad 3947.7 \times 10^3 = l \times 40 \times 42 \times \frac{80}{2}$$

$$\therefore \quad l = 58.74 \ \ mm$$

Homework:

1- A shaft running at (400 r.p.m.) transmits (10 kW). Assuming allowable shear stress in shaft as (40 MPa), find the diameter of the shaft.

2- A solid shaft is transmitting (1 MW) at (240 r.p.m.). Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as (60 MPa).

3- A hollow shaft for a rotary compressor is to be designed to transmit a maximum torque of (4750 N-m). The shear stress in the shaft is limited to (50 MPa). Determine the inside and outside diameters of the shaft, if the ratio of the inside to the outside diameter is (0.4).

4- A hollow shaft for a rotary compressor is to be designed to transmit a maximum torque of (4750 N-m). The shear stress in the shaft is limited to (50 MPa). Determine the inside and outside diameters of the shaft, if the ratio of the inside to the outside diameter is (0.4).

وزارة التعليم العالي والبحث العلمي

هيئة التعليم التقني

المعهد التقني – الديوانية

حقيبة رقم (7)

وحده نمطية لدراسة (*Power Screws*)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة لوالب نقل القدرة وانواعها وحساباتها

إعداد

المدرس

فائق حامد جبوري

CHAPTER (10)

Power screws

10 – 1 Introduction

The power screws (also known as translation screws) are used to convert rotary motion into translatory motion. For example, in the case of the lead screw of lathe, the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material. In case of screw jack, a small force applied in the horizontal plane is used to raise or lower a large load. Power screws are also used in vices, testing machines, presses, etc.

10 – 2 Types of screw threads used for power screw:

1- Square thread.

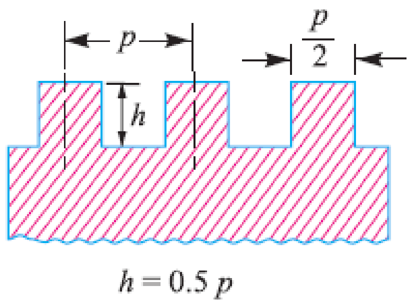


Fig. (10 – 1)

2- Acme thread.

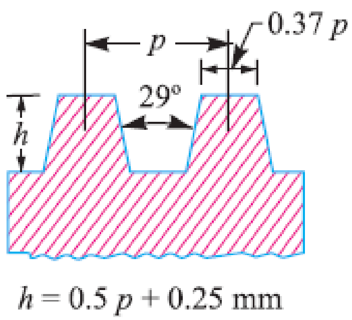


Fig. (10 – 2)

3- Buttress thread.

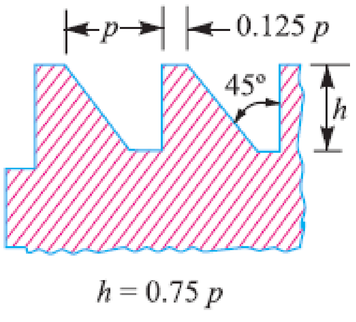


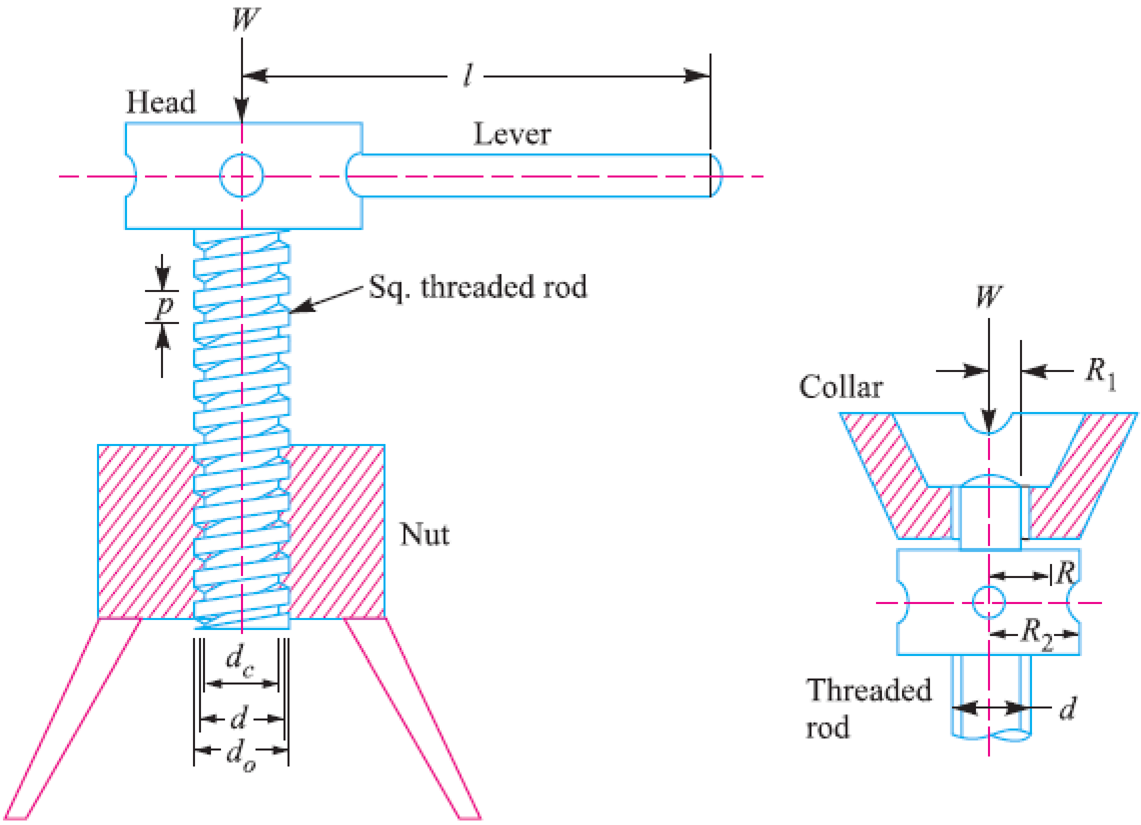
Fig. (10 – 3)

10 – 3 Multiple Threads

The power screws with multiple threads such as double, triple etc. are employed when it is desired to secure a large lead with fine threads or high efficiency. Such type of threads are usually found in high speed actuators.

10 – 4 Torque required to raise load on square threaded screws

The torque required to raise a load by means of square threaded screw may be determined by considering a screw jack as shown in Fig.(10 – 4 . *a*). The load to be raised or lowered is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of lever for lifting or lowering the load.



(*a*) Screw jack.

(*b*) Thrust collar.

Fig. (10 – 4)

A little consideration will show that if one complete turn of a screw thread be imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig. (10 – 4 . *a*).

p – Pitch of the screw.

D – Mean diameter of collar.

d – Mean diameter of the screw. $$d = \frac{d_o + d_c}{2} = d_o - \frac{p}{2} = d_c + \frac{p}{2}$$

d_o – Major diameter of the screw.

d_c – Core diameter of the screw.

α – Helix angle. $\tan\alpha = \dfrac{p}{\pi d}$

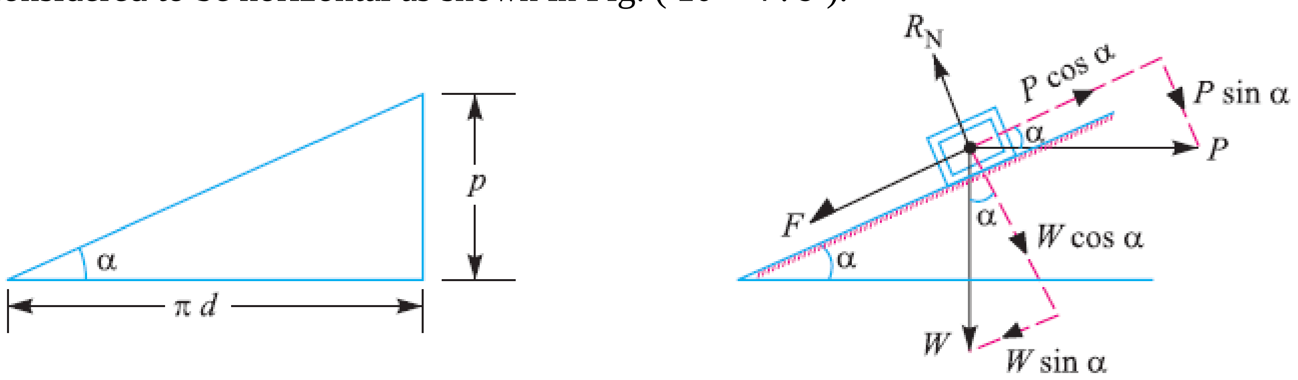
P – Effort applied at the circumference of screw to lift the load .
W – Weight of the body to be lifted.
μ_1 – Coefficient of friction for the collar.
μ - Coefficient of friction between the screw and nut.

$F = \mu \, . \, R_N$

$\mu = \tan\phi$

Where ϕ - is the friction angle.

Since the principle, on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the circumference of a screw jack may be considered to be horizontal as shown in Fig. (10 – 4 . *b*).



(*a*) Development of a screw. (*b*) Forces acting on the screw.

Fig. (10 – 5)

$$+\nwarrow \Sigma F_Y = 0$$

$$R_N = P\sin\alpha + W\cos\alpha$$

$$\overset{+}{\nearrow} \Sigma F_X = 0$$

$$P\cos\alpha = W\sin\alpha + F$$

$$= W\sin\alpha + \mu . R_N$$

$$= W\sin\alpha + \mu(P.\sin\alpha + W\cos\alpha)$$

$$= W\sin\alpha + \mu.P.\sin\alpha + \mu.W.\cos\alpha$$

$$P.\cos\alpha - \mu.P.\sin\alpha = W\sin\alpha + \mu.W.\cos\alpha$$

$$P(\cos\alpha - \mu.\sin\alpha) = W(\sin\alpha + \mu.\cos\alpha)$$

$$P = W \times \frac{\sin\alpha + \mu.\cos\alpha}{\cos\alpha - \mu.\sin\alpha}$$

$$P = W \times \frac{\sin\alpha + \tan\phi.\cos\alpha}{\cos\alpha - \tan\phi.\sin\alpha}$$

$$P = W \times \frac{\sin\alpha.\cos\phi + \sin\phi.\cos\alpha}{\cos\alpha.\cos\phi - \sin\phi.\sin\alpha}$$

$$= W \times \frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)}$$

$$= W \times \tan(\alpha+\phi) \qquad = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}\right]$$

T – Torque required to overcome friction between the screw and nut.

$$T = P \times \frac{d}{2}$$

$$\therefore\ T = W \times \tan(\alpha+\phi) \times \frac{d}{2}$$

10 – 5 Efficiency of Square Threaded Screws (η)

If there have been no friction between the screw and net, then (ϕ) will be equal to zero. The value of effort (P_o) necessary to raise the load, will then be given by the equation

$$P_o = W \times \tan\alpha \qquad\qquad \phi = 0$$

Efficiency:- $$\eta = \frac{Ideal\ effort}{Actual\ effort} = \frac{P_o}{P} = \frac{W\tan\alpha}{W\tan(\alpha+\phi)} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

This shows that efficiency of a screw jack, is independent of load raised. In the above expression for efficiency, only the screw friction is considered. However, if the screw friction and collar friction is taken into account, then

$$\eta = \frac{T_o}{T} = \frac{P_o \times d/2}{P \times d/2 + \mu_1 WR}$$

Where:- T_o = Torque required to move the load, neglecting friction.

T = Torque required to move the load, including screw and collar friction.

R = Mean radius of collar $R = \frac{R_1 + R_2}{2}$

<u>Example (1)</u>:- A vertical screw with single start square threads (50 mm) mean diameter and (12.5 mm) pitch is raised against a load of (10 kN) by means of hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a mean diameter of (60cm).The Coefficient of friction is (0.15) for the screw and (0.18) for the collar. If the tangential force applied by each hand to the wheel is (100 N). Find the suitable diameter of the hand wheel.

Solution:- Given: d = 50 mm, p = 12.5 mm, W= 10 KN = 10×10^3 N, D = 60 mm, $\mu = \tan\phi$ = 0.15 , μ_1 = 0.18 , F_{hand} = 100 N

$$\tan\alpha = \frac{p}{\pi d} = \frac{12.5}{\pi\times50} = 0.08$$

$$P = W\left[\frac{\tan\alpha+\tan\phi}{1-\tan\alpha.\tan\phi}\right]$$

$$=10\times10^3\left[\frac{0.08+0.15}{1-0.08\times0.15}\right] = 2328N$$

We also know that the total torque required to turn the hand wheel,

$$T = P\times\frac{d}{2}+\mu_1.W.R$$

$$=23280\times\frac{50}{2}+0.18\times10\times10^3\times30$$

$$=112200N.mm$$

Let $\quad D_1$ = Diameter of the hand wheel in mm.

We know that the torque applied to the hand wheel.

$$T = 2F_{hand}\times\frac{D_1}{2} \Rightarrow 112200 = 2\times100\frac{D_1}{2} = 100D_1$$

$$D_1 = \frac{112200}{100} = 1122mm \quad D_1 = 1.122m$$

<u>Example (2)</u>:- An electric motor driven power screw moves a nut a horizontal plane against a force of (75 kN) a speed of (300 mm / min.). The screw has a single square thread of (6 mm) pitch on a major diameter of (40 mm). The Coefficient of friction at screw threads is (0.1). Estimate power of the motor.

Solution:- Give: W= 75 kN = 75×10^3 N, v= 300 mm/min. , p= 6 mm, d_o = 40 mm, $\mu = \tan\phi = 0.1$

$$d = d_o - \frac{p}{2} \qquad = 40 - \frac{6}{2} = 37mm$$

$$\tan\alpha = \frac{p}{\pi d} \qquad = \frac{6}{\pi 37} = 0.0516$$

We know that tangential force required at the circumference of the screw,

$$P = W\tan(\alpha+\phi) = W\left[\frac{\tan\alpha + \tan\phi}{1-\tan\alpha.\tan\phi}\right]$$

$$P = 75\times10^3\left[\frac{0.0516+0.1}{1-0.0516\times0.1}\right] = 11.43\times10^3\,N$$

and torque required to operate the screw,

$$T = P\times\frac{d}{2} \qquad = 11.43\times\frac{37}{2} \qquad = 211.45 \text{ kN.mm}$$

$$N = \frac{Speed\,\frac{mm}{\min}}{Pitch\ mm} = \frac{300}{6} = 50rpm$$

and angular speed, $\omega = \frac{2\pi N}{60} = \frac{2\pi\times50}{60} = 5.24\,{}^{rad}\!/_{s}$

$\therefore$ Power of motor = $T\times\omega$

$$= 211.45\times5.24 = 1108W = 1.108kW$$

<u>Example (3)</u>:- The cutter of a broaching machine is pulled by square threaded screw of (55 mm) external diameter and (10 mm) pitch. The operating nut takes the axial load (400 N) on a flat surface of (60 mm) and (90 mm) internal and external diameters respectively. If the Coefficient of friction is (0.15) for all contact surface on the nut, determine the power required to rotate the operating nut when the cutting speed is (6 m/min). Also find the efficiency of the screw.

Solution:- Give – d_o= 55mm, p= 10 mm, W = 400 N, D_1 = 60 mm, D_2 = 90 mm, $\mu = \tan\phi = 0.15 = \mu_1$, Cutting speed = 6 m/min

Power required to operate the nut

We know that the mean diameter of the screw,

$$d = d_o - \frac{p}{2} \qquad d = 55 - \frac{10}{2} = 50mm$$

$$\tan\alpha = \frac{p}{\pi d} \qquad \tan\alpha = \frac{10}{\pi \times 50} = 0.0637$$

and force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}\right]$$

$$P = 400\left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15}\right] = 86.4 \;\; N$$

We know that mean radius of the flat surface,

$$R = \frac{R_1 + R_2}{2} = \frac{30 + 45}{2} = 37.5mm$$

Total torque required,

$$T = P \times \frac{d}{2} + \mu_1 WR \qquad = 86.4 \times \frac{50}{2} + 0.15 \times 400 \times 37.5$$

$$T = 4410 \;\; N.mm \qquad = 4.41 \;\; N.m$$

We know that speed of the screw, *N = Cutting speed / Pitch*

$$N = \frac{6}{0.01} = 600 \ rpm$$

and angular speed, $$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 600}{60} = 62.85 \ {}^{rad}\!/_{s}$$

$\therefore$ Power required operating the nut = $T \times \omega = 4.41 \times 62.84$

$$= 277W = 0.277kW$$

Efficiency of the screw $$\eta = \frac{T_o}{T} = \frac{W \tan\alpha \times {}^{d}\!/_{2}}{T}$$

$$\eta = \frac{400 \times 0.0637 \times {}^{50}\!/_{2}}{4410} = 14.4\%$$

Homework:

1- Prove that the Torque required to lower load by square threaded screws is:-

$$T = W \times \tan(\phi - \alpha) \times \frac{d}{2}$$

2- A vertical two start square threaded screw of a (100 mm) mean diameter and (20 mm) pitch supports a vertical load of (18 kN). The axial thrust on the screw is taken by a collar bearing of (250 mm) outside diameter and (100 mm) inside diameter. Find the force required at the end of a lever which is (400 mm) long in order to lift and lower the load. The coefficient of friction for the vertical screw and nut is (0.15) and that for collar bearing is (0.20).

وزارة التعليم العالي والبحث العلمي

جامعة الفرات الأوسط التقنية

المعهد التقني – الديوانية

حقيبة رقم (11)

وحده نمطية لدراسة (*Bearing And Lubricants*)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة كراسي المحاور والتزييت

(أنواعها, مجالات استخدامها, حسابات كراسي التحميل الإنزلاقية, التزييت, موانع التسرب)

إعداد

المدرس

فائق حامد جبوري

CHAPTER (11)

Bearing And Lubricants

11 – 1 Bearing

11 – 2 Introduction

A bearing is a machine element which support another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as lubricant) may be provided . The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicon oils, greases etc., may be used.

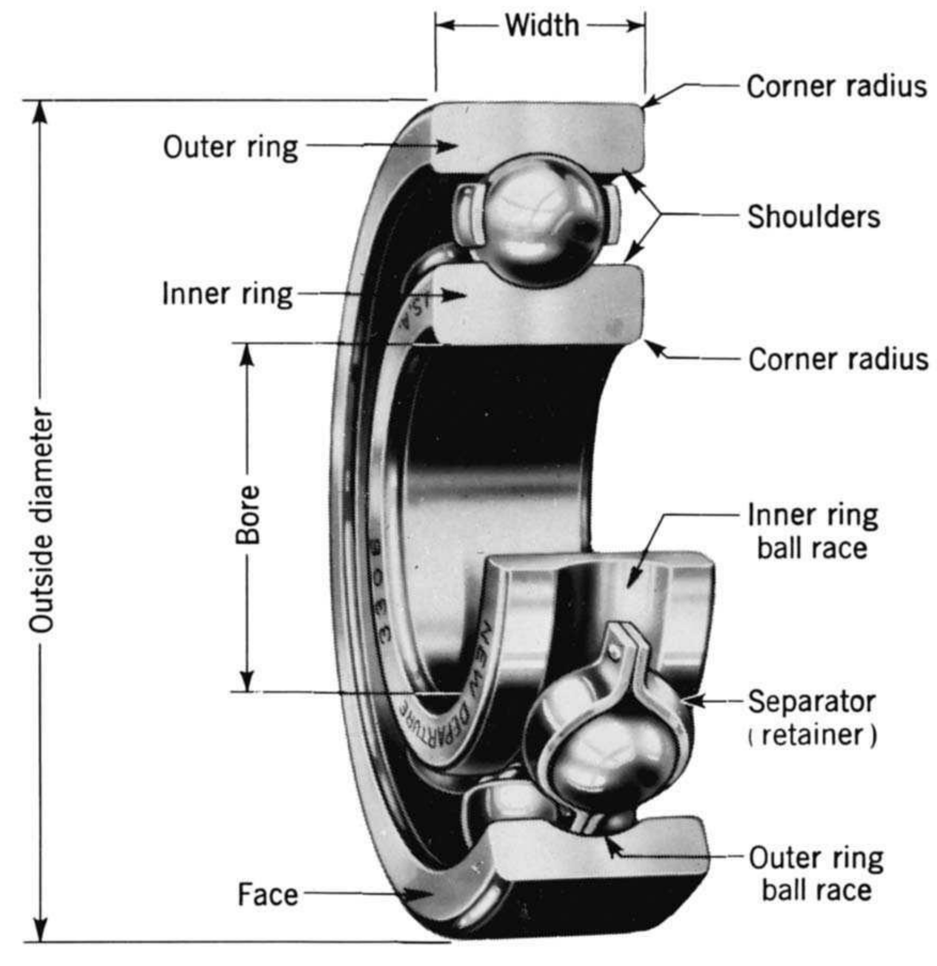


Fig. (11 – 1) Roller Bearing.

11 – 3 Classification of Bearings

Though the bearings may be classified in many ways, yet the following are important from the subject point of view:

1- Depending upon the direction of load to be supported

The bearings under this group are classified as:

a- Radial bearings:- In radial bearings, the load acts perpendicular to the direction of motion of the moving element as shown in Fig. (11 – 2).

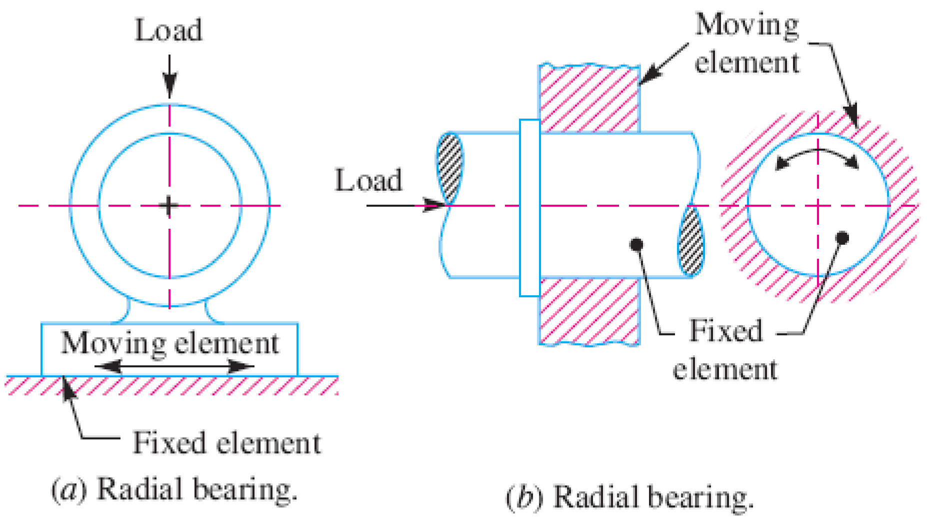


Fig. (11 – 2). Radial Bearing.

b- Thrust bearings:- In thrust bearings, the load acts along the axis of rotation as shown in Fig. (11 – 3).

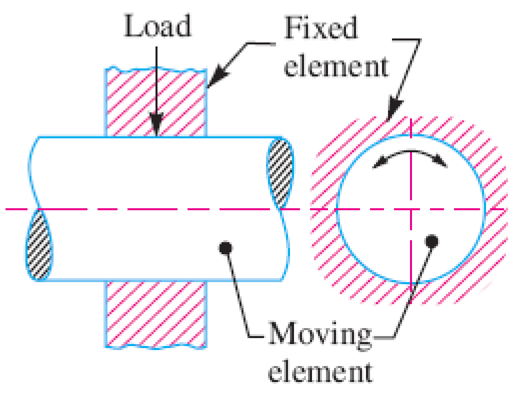


Fig. (11 – 3). Thrust bearing.

2- Depending upon the nature of contact.

The bearings under this group are classified as:

a- Sliding contact bearings:- In sliding contact bearings, as shown in Fig. (11–4), the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as plain bearings.

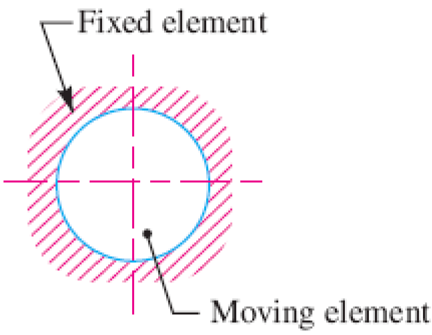


Fig. (11 – 4) Sliding contact bearing.

b- Rolling contact bearings:- In rolling contact bearings, as shown in Fig. (11 – 5), the steel balls or rollers, are interposed between the moving and fixed elements. The balls offer rolling friction at two points for each ball or roller.

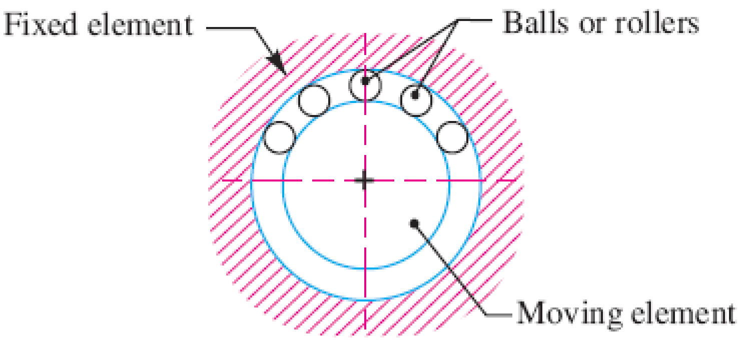


Fig. (11 – 5) Rolling contact bearings.

11 – 4 Types of Sliding Contact Bearings

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, as shown in Fig. (11 – 2), may be called **slipper** or **guide bearings**. Such type of bearings are usually found in cross-head of steam engines. The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as **journal** or **sleeve bearings.**

1- full journal bearing:- When the angle of contact of the bearing with the journal is (360°) as shown in Fig. (11 – 6), then the bearing is called a **full journal bearing.** This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction.

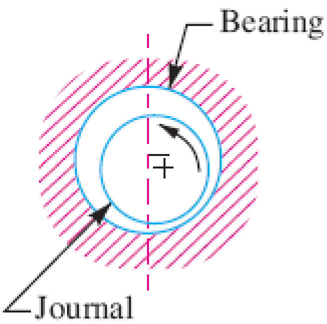


Fig. (11 – 6) Full journal bearing.

2- Partial journal bearing:-When the angle of contact of the bearing with the journal is (120°), as shown in Fig. (11 – 7), then the bearing is said to be **partial journal bearing.** This type of bearing has less friction than full journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings may be called as **clearance bearings** because the diameter of the journal is less than that of bearing.

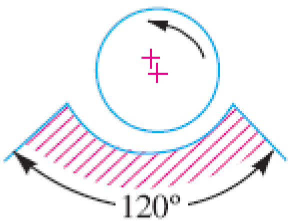


Fig. (11 – 7) Partial journal bearing.

3- Fitted journal bearing:- When a partial journal bearing has no clearance i.e. the diameters of the journal and bearing are equal, then the bearing is called a **fitted bearing,** as shown in Fig. (11 – 8). The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal may also be classified as follows:

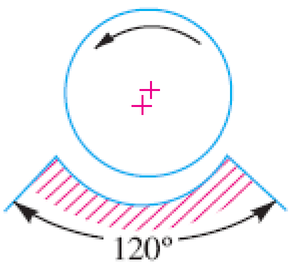


Fig. (11 – 8) Fitted journal bearing.

a- Thick film bearings:- The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as **hydrodynamic lubricated bearings.**

b- Thin film bearings:- The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other at least part of the time. Such type of bearings are also called **boundary lubricated bearings.**

c- Zero film bearings:- The zero film bearings are those which operate without any lubricant present.
d- Hydrostatic or externally pressurized lubricated bearings:- The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members.

11 – 5 Properties of Sliding Contact Bearing Materials

When the journal and the bearings are having proper lubrication *i.e.* there is a film of clean, non-corrosive lubricant in between, separating the two surfaces in contact, the only requirement of the bearing material is that they should have sufficient strength and rigidity. However, the conditions under which bearings must operate in service are generally far from ideal and thus the other properties as discussed below must be considered in selecting the best material.

1- Compressive strength	2- Fatigue strength
3- Conformability	4- Embeddability
5- Bondability	6- Corrosion resistance
7- Thermal conductivity	8- Thermal expansion

11 – 6 Materials used for Sliding Contact Bearings

The materials commonly used for sliding contact bearings are discussed below:

1- Babbit metal:- The tin base and lead base babbits are widely used as a bearing material, because they satisfy most requirements for general applications. The babbits are recommended where the maximum bearing pressure (on projected area) is not over 7 to 14 N/mm2. When applied in automobiles, the babbit is generally used as a thin layer, 0.05 mm to 0.15 mm thick, bonded to an insert or steel shell. The composition of the babbit metals is as follows:
Tin base babbits : Tin 90% , Copper 4.5% , Antimony 5% , Lead 0.5%.
Lead base babbits : Lead 84% , Tin 6% , Antimony 9.5% , Copper 0.5%.
2- Bronzes:- The bronzes (alloys of copper, tin and zinc) are generally used in the form of machined bushes pressed into the shell. The bush may be in one or two pieces. The bronzes commonly used for bearing material are gun metal and phosphor bronzes.
3- Cast iron:- The cast iron bearings are usually used with steel journals. Such type of bearings are fairly successful where lubrication is adequate and the pressure is limited to 3.5 N/mm2 and speed to 40 metres per minute.
4- Silver:- The silver and silver lead bearings are mostly used in aircraft engines where the fatigue strength is the most important consideration.
5- Non-metallic bearings:- The various non-metallic bearings are made of carbon-graphite, rubber, wood and plastics.

11- 7 Lubricants

The lubricants are used in bearings to reduce friction between the rubbing surfaces and to carry away the heat generated by friction. It also protects the bearing against corrosion. All lubricants are classified into the following three groups :

1- Liquid:- The liquid lubricants usually used in bearings are mineral oils and synthetic oils. The mineral oils are most commonly used because of their cheapness and stability. The liquid lubricants are usually preferred where they may be retained.

2- Semi-liquid:- A grease is a semi-liquid lubricant having higher viscosity than oils. The greases are employed where slow speed and heavy pressure exist and where oil drip from the bearing is undesirable.

3- Solid:- The solid lubricants are useful in reducing friction where oil films cannot be maintained because of pressures or temperatures. They should be softer than materials being lubricated. A graphite is the most common of the solid lubricants either alone or mixed with oil or grease.

11 – 8 Properties of Lubricants

1- Viscosity:- It is the measure of degree of fluidity of a liquid. It is a physical property by virtue of which an oil is able to form, retain and offer resistance to shearing a buffer film-under heat and pressure. The greater the heat and pressure, the greater viscosity is required of a lubricant to prevent thinning and squeezing out of the film.

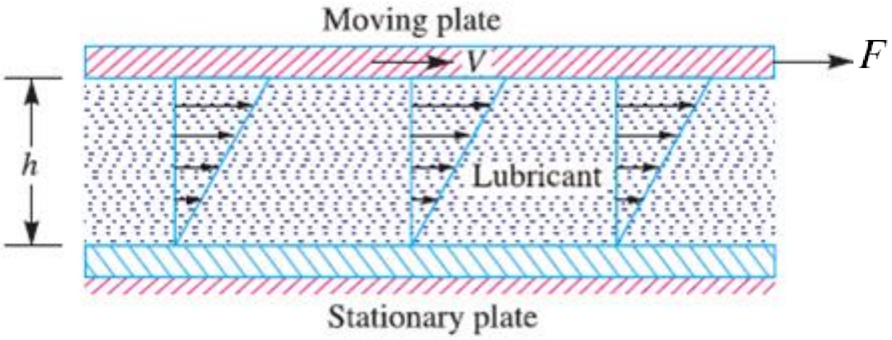


Fig. (11 – 9)

$$\tau = \frac{F}{A}$$

According to Newton's law of viscous flow, the magnitude of this shear stress varies directly with the velocity gradient (dV / dy). It is assumed that

a- the lubricant completely fills the space between the two surfaces,

b- the velocity of the lubricant at each surface is same as that of the surface, and

c- any flow of the lubricant perpendicular to the velocity of the plate is negligible.

$$\therefore \ \tau = \frac{F}{A} \ \alpha \ \frac{dV}{dy} \quad \text{or} \quad \tau = Z \times \frac{dV}{dy}$$

Where (Z) is a constant of proportionality and is known as **absolute viscosity** (or simply viscosity) of the lubricant.
When the thickness of the fluid lubricant is small which is the case for bearings, then the velocity gradient is very nearly constant as shown in Fig. (11 – 9), so that

$$\frac{dV}{dy} = \frac{V}{y} = \frac{V}{h}$$

$$\therefore \ \tau = Z \times \frac{V}{h} \quad \text{or} \quad Z = \tau \times \frac{h}{V}$$

When:- τ is in (N / m^2), h is in (metres) and V is in (m / s),

then the unit of absolute viscosity is given by

$$Z = \tau \times \frac{h}{V} \quad = \frac{N}{m^2} \times \frac{m}{m/s} = \frac{N.\ s}{m^2}$$

However, the common practice is to express the absolute viscosity in mass units, such that

$$\frac{N.\ s}{m^2} = \frac{1\ kg.\ m}{s^2} \times \frac{s}{m^2} = 1\ kg/m.\ s$$

Thus the unit of absolute viscosity in S.I. units is kg / m-s.

2- Oiliness:- It is a joint property of the lubricant and the bearing surfaces in contact. It is a measure of the lubricating qualities under boundary conditions where base metal to metal is prevented only by absorbed film. There is no absolute measure of oiliness.

3- Density:- This property has no relation to lubricating value but is useful in changing the kinematic viscosity to absolute viscosity. Mathematically

Absolute viscosity = ρ × Kinematic viscosity (in m^2/s)
Where:- ρ = Density of the lubricating oil.

4- Viscosity index:- The term viscosity index is used to denote the degree of variation of viscosity with temperature.

5- Flash point:- It is the lowest temperature at which an oil gives off sufficient vapour to support a momentary flash without actually setting fire to the oil when a flame is brought within 6 mm at the surface of the oil.

6- Fire point:- It is the temperature at which an oil gives off sufficient vapour to burn it continuously when ignited.
7- Pour point or freezing point:- It is the temperature at which an oil will cease to flow when cooled.

11 – 9 Terms used in Hydrodynamic Journal Bearing

A hydrodynamic journal bearing is shown in Fig. (11 – 10), in which *O* is the centre of the journal
and *O′* is the centre of the bearing.
Let D = Diameter of the bearing,
d = Diameter of the journal, and
l = Length of the bearing.
The following terms used in hydrodynamic journal bearing are important from the subject point of view :

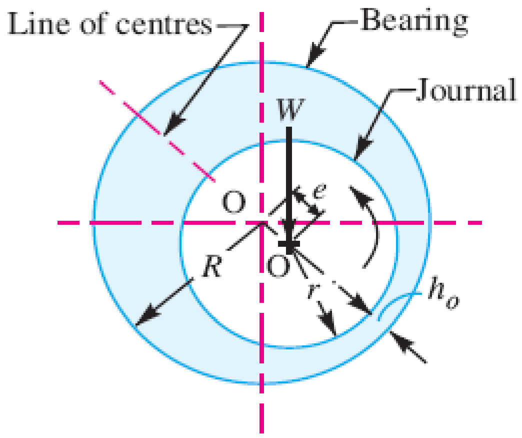


Fig. (11 – 10). Hydrodynamic journal bearing.

1- Diametral clearance (*c*):- It the difference between the diameters of the bearing and the journal. Mathematically, diametral clearance,

$$c = D - d$$

Note : The diametral clearance (*c*) in a bearing should be small enough to produce the necessary velocity gradient, so that the pressure built up will support the load. Also the small clearance has the advantage of decreasing side leakage. However, the allowance must be made for manufacturing tolerances in the journal and bushing. A commonly used clearance in industrial machines is (0.025 mm per cm) of journal diameter.

2- Radial clearance (c_1): It is the difference between the radii of the bearing and the journal. Mathematically, radial clearance,

$$c_1 = R - r = \frac{D-d}{2} = \frac{c}{2}$$

3- Diametral clearance ratio (c / d):- It is the ratio of the diametral clearance to the diameter of the journal. Mathematically, diametral clearance ratio

$$\frac{c}{d} = \frac{D-d}{d}$$

4- Eccentricity (e):- It is the radial distance between the centre (O) of the bearing and the displaced centre (O′) of the bearing under load. It is denoted by (e).

5- Minimum oil film thickness. It is the minimum distance between the bearing and the journal, under complete lubrication condition. It is denoted by h0 and occurs at the line of centres as shown in Fig. (11 – 10). Its value may be assumed as (c / 4).

6- Attitude or eccentricity ratio (ε):- It is the ratio of the eccentricity to the radial clearance. Mathematically, attitude or eccentricity ratio,

$$\varepsilon = \frac{e}{c_1} = \frac{c_1 - h_o}{c_1} = 1 - \frac{h_o}{c_1}$$

$$\because \quad c_1 = \frac{c}{2}$$

$$\therefore \quad \varepsilon = 1 - \frac{2h_o}{c}$$

7- Short and long bearing. If the ratio of the length to the diameter of the journal (i.e. l / d) is less than (1), then the bearing is said to be **short bearing.**
On the other hand, if (l / d) is greater than (1), then the bearing is known as **long bearing.**

Notes:-

1- When the length of the journal (l) is equal to the diameter of the journal (d), then the bearing is called **square bearing.**

2- Because of the side leakage of the lubricant from the bearing, the pressure in the film is atmospheric at the ends of the bearing. The average pressure will be higher for a long bearing than for a short or square bearing. Therefore, from the stand point of side leakage, a bearing with a large (l / d) ratio is preferable. However, space requirements, manufacturing, tolerances and shaft deflections are better met with a short bearing. The value of (l / d) may be taken as (1 to 2) for general industrial machinery. In crank shaft bearings, the (l / d) ratio is frequently less than (1).

11 – 10 Bearing Characteristic Number and Bearing Modulus for Journal Bearings

The coefficient of friction in design of bearings is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal bearing is a function of three variables, *i.e.*

1- $\frac{ZN}{P}$ 2- $\frac{d}{c}$ 3- $\frac{l}{d}$

Therefore the coefficient of friction may be expressed as

$$\mu = \phi \left(\frac{ZN}{P} \ , \ \frac{d}{c} \ , \ \frac{l}{d} \right)$$

Where:- μ = Coefficient of friction,

ϕ = A functional relationship,

Z = Absolute viscosity of the lubricant, in kg / m-s,

N = Speed of the journal in r.p.m.,

P = Bearing pressure on the projected bearing area in N/mm^2,

$P = \frac{W}{l \times d}$ W = Load on the journal

d = Diameter of the journal,

l = Length of the bearing, and

c = Diametral clearance.

The factor (ZN / P) is termed as **bearing characteristic number** and is a dimensionless number.

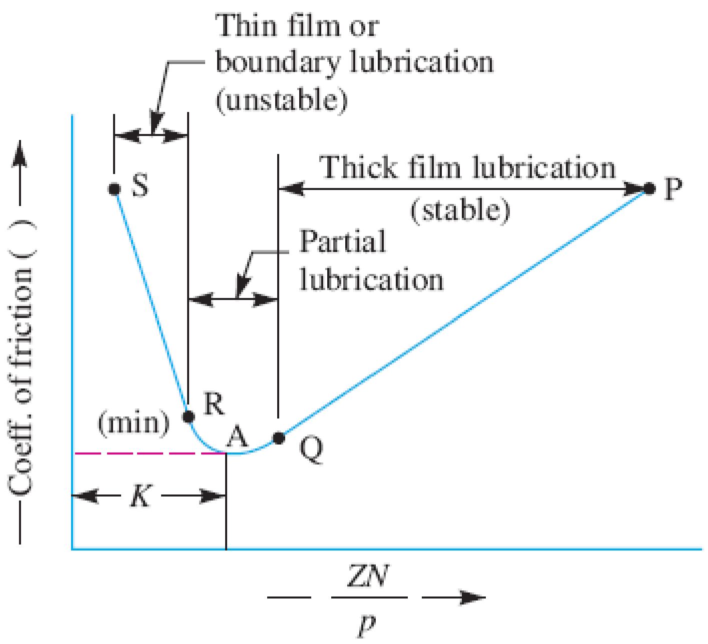


Fig. (11 – 11). Variation of coefficient of friction with *Z.N / P*.

From Fig. (11 – 11), we see that the minimum amount of friction occurs at (A)and at this point the value of (*ZN / P*) is known as **bearing modulus** which is denoted by (K). The bearing should not be operated at this value of bearing modulus, because a slight decrease in speed or slight increase in pressure will break the oil film and make the journal to operate with metal to metal contact. This will result in high friction, wear and heating. In order to prevent such conditions, the bearing should be designed for a value of (*ZN / P*) at least three times the minimum value of bearing modulus (*K*). If the bearing is subjected to large fluctuations of load and heavy impacts, the value of (*ZN / P = 15 K*)may be used.

From above, it is concluded that when the value of (*ZN / P*) is greater than (*K*), then the bearing will operate with thick film lubrication or under hydrodynamic conditions. On the other hand, when the value of (*ZN / P*) is less than (*K*), then the oil film will rupture and there is a metal to metal contact.

11 – 11 Coefficient of Friction for Journal Bearings

$$\mu = \frac{33}{10^8}\left(\frac{Z.N}{P}\right)\left(\frac{d}{c}\right) + k$$ (when Z is in kg / m-s and P is in N / mm^2)

Where:- Z , N , P , d and c have usual meanings as discussed in previous article,
and k = Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (*i.e. l / d*).
= 0.002 for (*l / d*) ratios of (0.75 to 2.8).

11 – 12 Heat Generated in a Journal Bearing

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing,

$$Q_g = \mu \times W \times V \qquad \text{----------------- (1)}$$

$$Q_g = N - m/s \quad or \quad J/s \quad Watts$$

Where:- μ = Coefficient of friction,
W = Load on the bearing in N,

$W = P \times (l \times d)$ ($l \times d$) Projected area of the bearing in (mm^2)

V = Rubbing velocity in m/s $= \dfrac{\pi \times d \times N}{60}$ d is in metres, and

N = Speed of the journal in r.p.m.

After the thermal equilibrium has been reached, heat will be dissipated at the outer surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat dissipated will depend upon the temperature difference, size and mass of the radiating surface and on the amount of air flowing around the bearing. However, for the convenience in bearing design, the actual heat dissipating area may be expressed in terms of the projected area of the journal.

Heat dissipated by the bearing (Q_d),

$$Q_d = C \times A \times (T_b - T_a) \qquad \text{----------------- (2)}$$

$$Q_d = J/s \quad or \quad W$$

Where:- C = Heat dissipation coefficient in W/m^2.°C,
A = Projected area of the bearing in m^2 $= l \times d$,
T_b = Temperature of the bearing surface in °C, and
T_a = Temperature of the surrounding air in °C.

It has been shown by experiments that the temperature of the bearing (T_b) is approximately mid-way between the temperature of the oil film (T_o) and the temperature of the outside air (T_a). In other words,

$$T_b - T_a = \frac{1}{2}(T_o - T_a)$$

Amount of artificial cooling required

$$Heat\ generation - Heat\ dissipated = Q_g - Q_d$$

Example (1):-The load on the journal bearing is (150 kN) due to turbine shaft of (300 mm) diameter running at (1800 r.p.m.) Determine the following:
1- Length of the bearing if the allowable bearing pressure is (1.6 N/mm^2), and
2- Amount of heat to be removed by the lubricant per minute if the bearing temperature is (60°C) and viscosity of the oil at (60°C is 0.02 kg/m-s) and the bearing clearance is (0.25 mm).
Solution:- Given: W = 150 kN = 150 × 103 N , d = 300 mm = 0.3 m ,
N = 1800 r.p.m. , P = 1.6 N/mm^2 ; Z = 0.02 kg / m-s ; c = 0.25 mm

1- *Length of the bearing*

$$A = l \times b = l \times 300 = 300 \times l \quad mm^2$$

and allowable bearing pressure (P),

$$P = \frac{W}{A} \Rightarrow 1.6 = \frac{150 \times 10^3}{300 \times l} = \frac{500}{l}$$

$\therefore\ l = 312.5\ mm$ **Ans.**

2- *Amount of heat to be removed by the lubricant*
We know that coefficient of friction for the bearing,

$$\mu = \frac{33}{10^8}\left(\frac{Z.N}{P}\right)\left(\frac{d}{c}\right) + k$$

$$\mu = \frac{33}{10^8}\left(\frac{0.02 \times 1800}{1.6}\right)\left(\frac{300}{0.25}\right) + 0.002 = 0.011$$

Rubbing velocity

$$V = \frac{\pi \times d \times N}{60} = \frac{\pi \times 0.3 \times 1800}{60} = 28.274\ m/s$$

$\therefore$ Amount of heat to be removed by the lubricant,

$$Q_g = \mu \times W \times V$$

$$Q_g = 0.011 \times 150 \times 10^3 \times 28.274 = 46.652\ kW$$ **Ans.**

Example (2):- A (150 mm) diameter shaft supporting a load of (10 kN) has a speed of (1500 r.p.m.) The shaft runs in a bearing whose length is (1.5) times the shaft diameter. If the diametral clearance of the bearing is (0.15 mm) and the absolute viscosity of the oil at the operating temperature is (0.011 kg/m-s), find the power wasted in friction.

Solution:- Given : d = 150 mm = 0.15 m , W = 10 kN = 10 000 N , N =1500 r.p.m. , l = 1.5 d , c = 0.15 mm , Z = 0.011 kg/m-s

We know that length of bearing,

$$l = 1.5 \times d = 1.5 \times 150 = 225 \ mm$$

$\therefore$ Bearing pressure,

$$P = \frac{W}{A} = \frac{W}{l \times d} = \frac{10000}{225 \times 150} = 0.296 \ N/mm^2$$

$$\mu = \frac{33}{10^8}\left(\frac{Z.N}{P}\right)\left(\frac{d}{c}\right) + k$$

$$\mu = \frac{33}{10^8}\left(\frac{0.011 \times 1500}{0.296}\right)\left(\frac{150}{0.15}\right) + 0.002 = 0.02$$

and rubbing velocity,

$$V = \frac{\pi \times d \times N}{60} = \frac{\pi \times 0.15 \times 1500}{60} = 11.78 \ m/s$$

We know that Power wasted in friction (heat generated due to friction) ,

$$Q_g = \mu \times W \times V$$

$$Q_g = 0.02 \times 10 \times 10^3 \times 11.78 = 2356 \ W \quad = 2.356 \ kW$$ **Ans.**

Homework:

1- A full journal bearing of (50 mm) diameter and (100 mm) long has a bearing pressure of (1.4 N/mm^2). The speed of the journal is (900 r.p.m.) and the ratio of journal diameter to the diametral clearance is (1000). The bearing is lubricated with oil whose absolute viscosity at the operating temperature of (75°C) may be taken as (0.011 kg/m-s). The room temperature is (35°C). Find the amount of artificial cooling required. Take C = 280 W/m^2 .° C

2- A journal bearing (60 mm) is diameter and (90 mm) long runs at (450 r.p.m.) The oil used for hydrodynamic lubrication has absolute viscosity of (0.06 kg /m-s). If the diametral clearance is (0.1 mm), find the safe load on the bearing.

3- A (80 mm) long journal bearing supports a load of (2800 N) on a (50 mm) diameter shaft. The bearing has a radial clearance of (0.05 mm) and the viscosity of the oil is (0.021 kg / m-s) at the operating temperature. If the bearing is capable of dissipating (80 J/s), determine the maximum safe speed.

وزارة التعليم العالي والبحث العلمي

جامعة الفرات الأوسط التقنية

المعهد التقني – الديوانية

حقيبة رقم (12)

وحده نمطية لدراسة (*Worm Gear*)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة اللولب والصامولة (الترس الدودي ودوره في توصيل الحركة من وإلى صناديق السرعات والتغذيات في المكائن الإنتاجية)

إعداد

المدرس

فائق حامد جبوري

CHAPTER (12)

Worm Gear

12 – 1 Introduction

The worm gears are widely used for transmitting power at high velocity ratios between non-intersecting shafts that are generally, but not necessarily, at right angles. It can give velocity ratios as high as (300 : 1) or more in a single step in a minimum of space, but it has a lower efficiency. The worm gearing is mostly used as a speed reducer, which consists of worm and a worm wheel or gear.

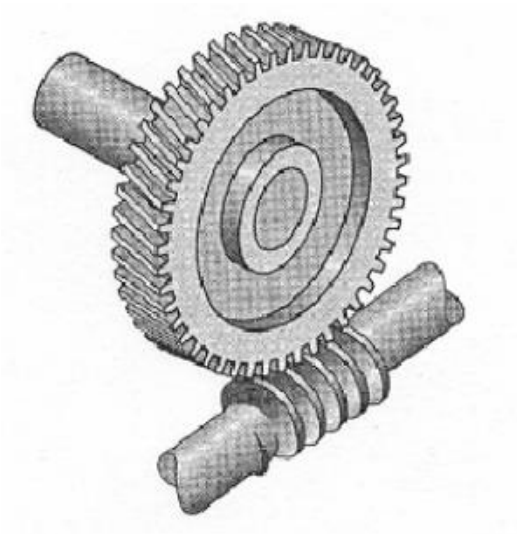


Fig. (12 – 1) Worm gear.

12 -2 Types of Worms

The following are the two types of worms :

1- Cylindrical or straight worm

The **cylindrical** or **straight worm**, as shown in Fig. (12 – 2 .a), is most commonly used. The shape of the thread is involute helicoid of pressure angle (14 ½°) for single and double threaded worms and (20°) for triple and quadruple threaded worms. The worm threads are cut by a straight sided milling cutter having its diameter not less than the outside diameter of worm or greater than (1.25) times the outside diameter of worm.

2- Cone or double enveloping worm

The **cone** or **double enveloping worm**, as shown in Fig. (12 – 2 . b), is used to some extent, but it requires extremely accurate alignment.

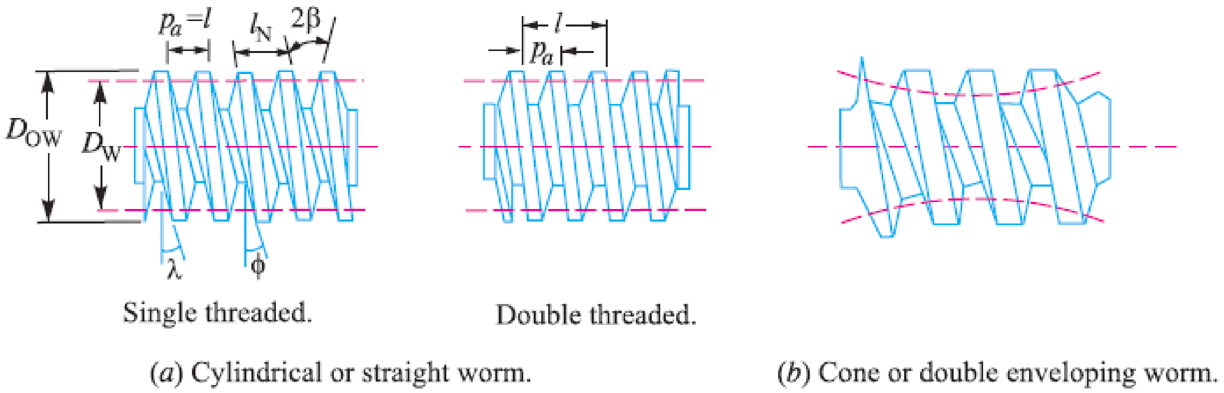


Fig. (12 – 2). Types of worms.

12 – 3 Types of Worm Gears

The following three types of worm gears are important from the subject point of view:

1- Straight face worm gear, as shown in Fig. (12 – 3.a),
2- Hobbed straight face worm gear, as shown in Fig. (12 – 3.b), and
3- Concave face worm gear, as shown in Fig. (12 – 3 .c).

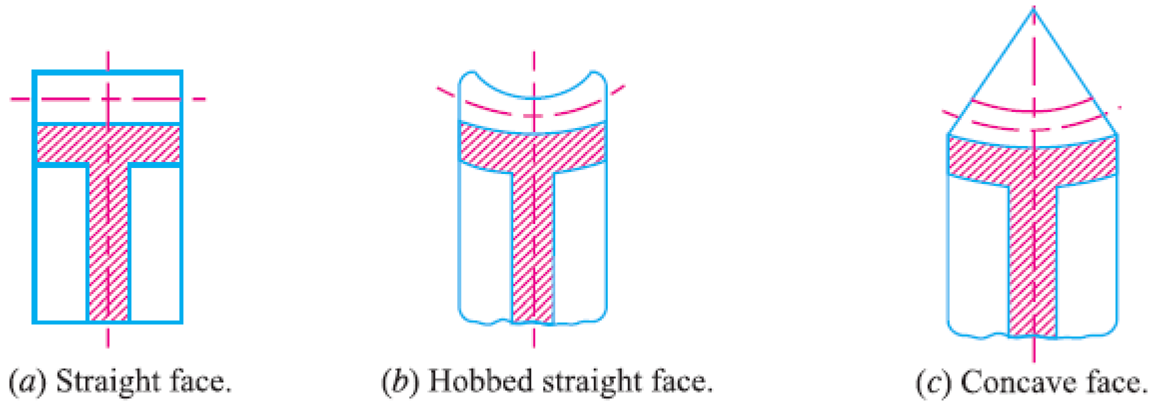


Fig. (12 – 3) Types of worms gears.

The **straight face worm gear** is like a helical gear in which the straight teeth are cut with a form cutter. Since it has only point contact with the worm thread, therefore it is used for light service.

The **hobbed straight face worm gear** is also used for light service but its teeth are cut with a hob, after which the outer surface is turned.

The **concave face worm gear** is the accepted standard form and is used for all heavy service and general industrial uses. The teeth of this gear are cut with a hob of the same pitch diameter as the mating worm to increase the contact area.

Fig. (12 – 4)

Worm gear is used mostly where the power source operates at a high speed and output is at a slow speed with high torque. It is also used in some cars and trucks.

وزارة التعليم العالي والبحث العلمي

هيئة التعليم التقني

المعهد التقني – الديوانية

حقيبة رقم (13)

وحده نمطية لدراسة (*The Cams*)

لطلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الفئة المستهدفة

طلبة المرحلة الثانية في قسم الميكانيك فرعي (التشغيل والصيانة – الإنتاج)

الهدف من الوحدة

دراسة الحدبات (الكامات)

(أنواعها , استخداماتها في المكائن الإنتاجية)

إعداد

المدرس

فائق حامد جبوري

CHAPTER (13)

The Cams

Introduction

A cam is a rotating machine element which gives reciprocating or oscillating motion to another element known as follower. The cam and the follower have a line contact and constitute a higher pair. The cams are usually rotated at uniform speed by a shaft, but the follower motion is predetermined and will be according to the shape of the cam. The cam and follower is one of the simplest as well as one of the most important mechanisms found in modern machinery today. The cams are widely used for operating the inlet and exhaust valves of internal combustion engines, automatic attachment of machineries, paper cutting machines, spinning and weaving textile machineries, feed mechanism of automatic lathes etc.

Classification of Followers

The followers may be classified as discussed below :

1- According to the surface in contact

The followers, according to the surface in contact, are as follows:-

a-Knife edge follower

It is seldom used in practice because the small area of contacting surface results in excessive wear. In knife edge followers, a considerable side thrust exists between the follower and the guide.

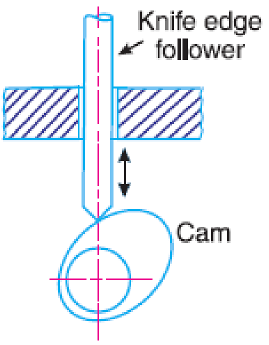


Fig. (13 – 1) Knife edge follower.

b- Roller follower

The roller followers are extensively used where more space is available such as in stationary gas and oil engines and aircraft engines.

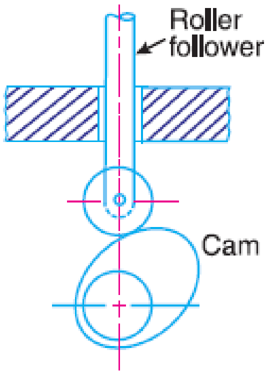


Fig. (13 – 2) Roller follower.

c- Flat faced or mushroom follower

The flat faced followers are generally used where space is limited such as in cams which operate the valves of automobile engines.

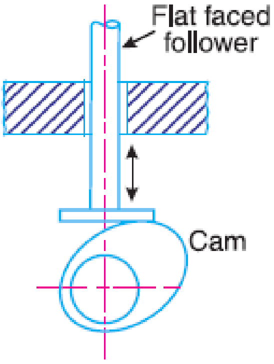


Fig. (13 – 3) Flat faced or mushroom follower.

d- Spherical faced follower

It may be noted that when a flat-faced follower is used in automobile engines, high surface stresses are produced. In order to minimise these stresses, the flat end of the follower is machined to a spherical shape.

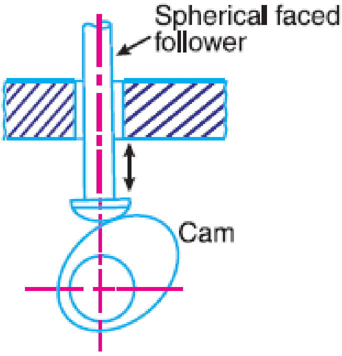


Fig. (13 – 4) Spherical faced follower.

2- According to the motion of the follower

The followers, according to its motion, are of the following two type:

a- Reciprocating or translating follower

The followers as shown in Fig. 1- (a) to (d) are all reciprocating or translating followers.

b- Oscillating or rotating follower

When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower.

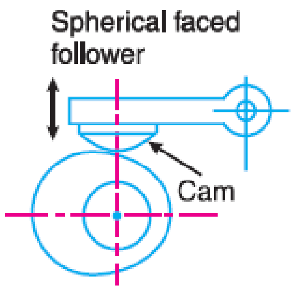


Fig. (13 – 5) Oscillating or rotating follower

3- According to the path of motion of the follower

The followers, according to its path of motion, are of the following two types:

a- Radial follower

When the motion of the follower is along an axis passing through the centre of the cam, it is known as radial follower. The followers, as shown in Fig. (a) to (d), and 2- (b) are all radial followers.

b- Off-set follower

When the motion of the follower is along an axis away from the axis of the cam centre, it is called off-set follower.

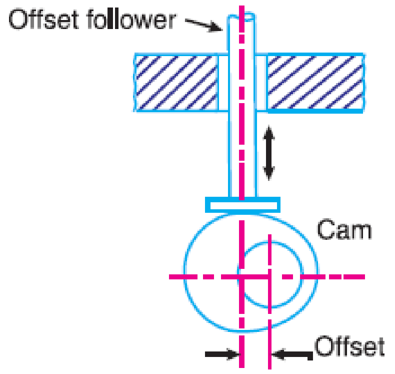


Fig. (13 – 5) Off-set follower.

Note:- In all cases, the follower must be constrained to follow the cam. This may be done by springs, gravity or hydraulic means. In some types of cams, the follower may ride in a groove.

Classification of Cams

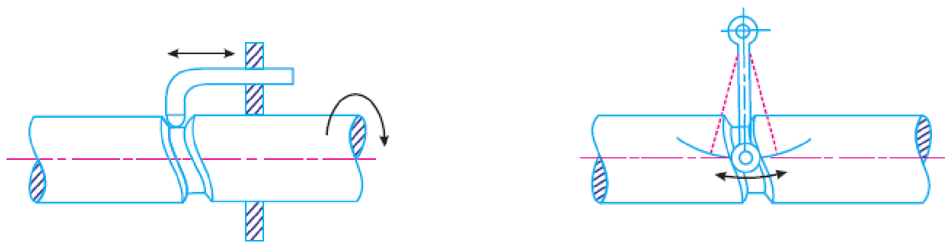
Though the cams may be classified in many ways, yet the following two types are important from the subject point of view:-

1- Radial or disc cam

In radial cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis. The cams as shown in last Figures are all radial cams.

2- Cylindrical cam

In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove at its cylindrical surface. A cylindrical grooved cam with a reciprocating and an oscillating follower is shown in Fig. below.



(a) Cylindrical cam with reciprocating follower. **(b) Cylindrical cam with oscillating follower.**

Fig. (13 – 6) Cylindrical cam.